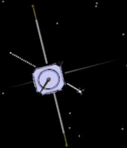
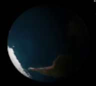


Flow-Informed Strategies for Trajectory Design and Analysis

Computational Science & Engineering Seminar

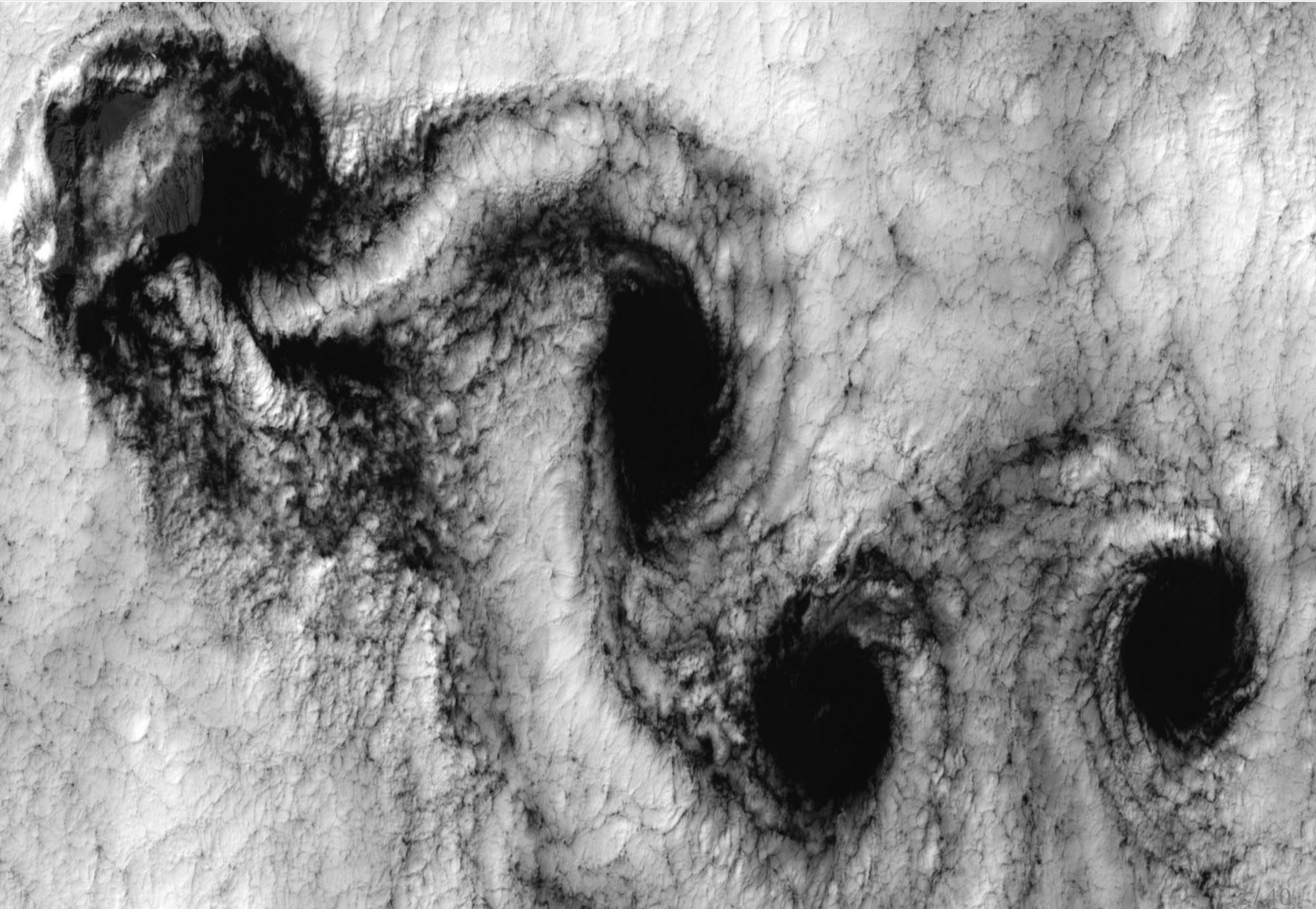
Cody R. Short



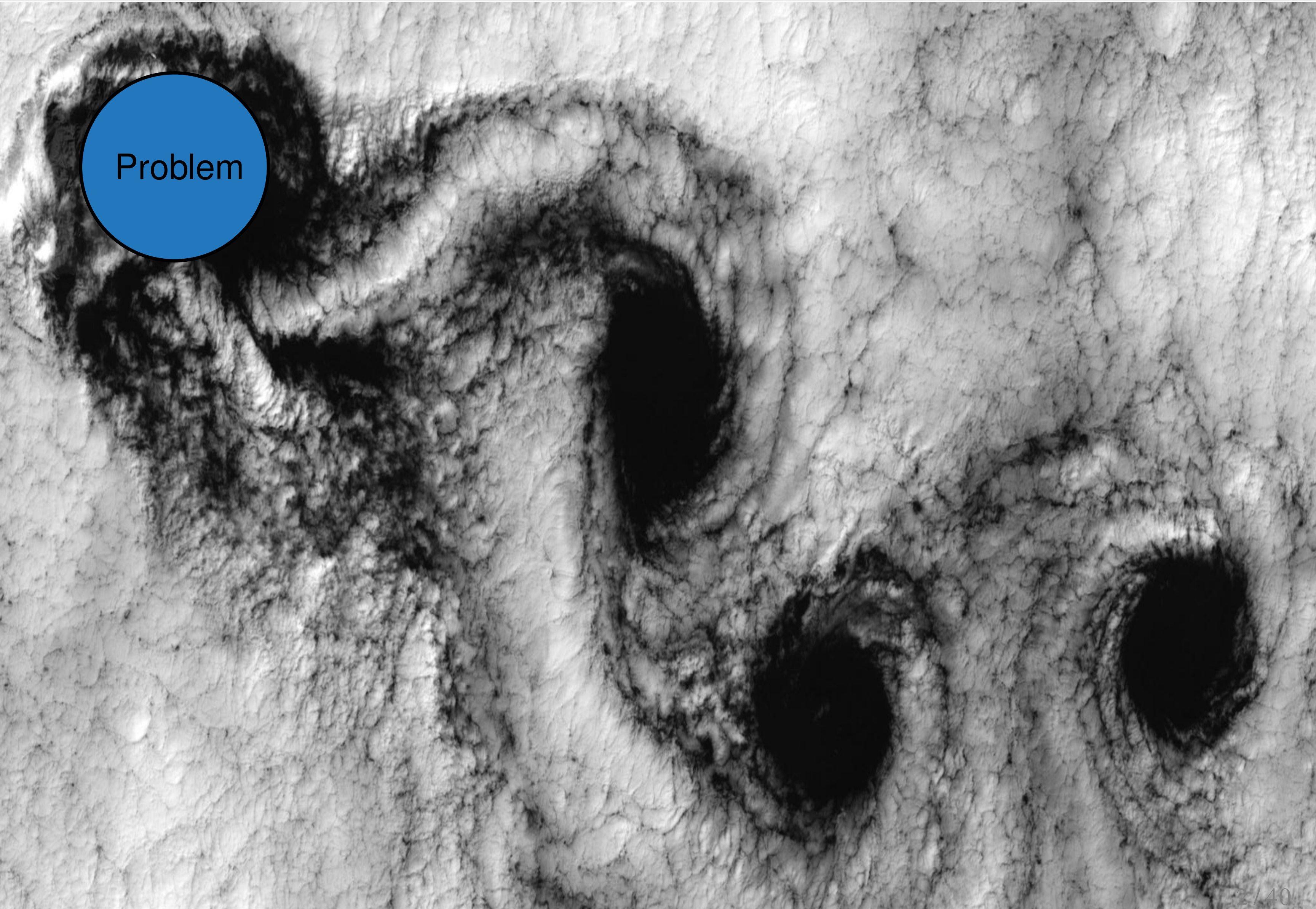
School of Aeronautics & Astronautics
Purdue University

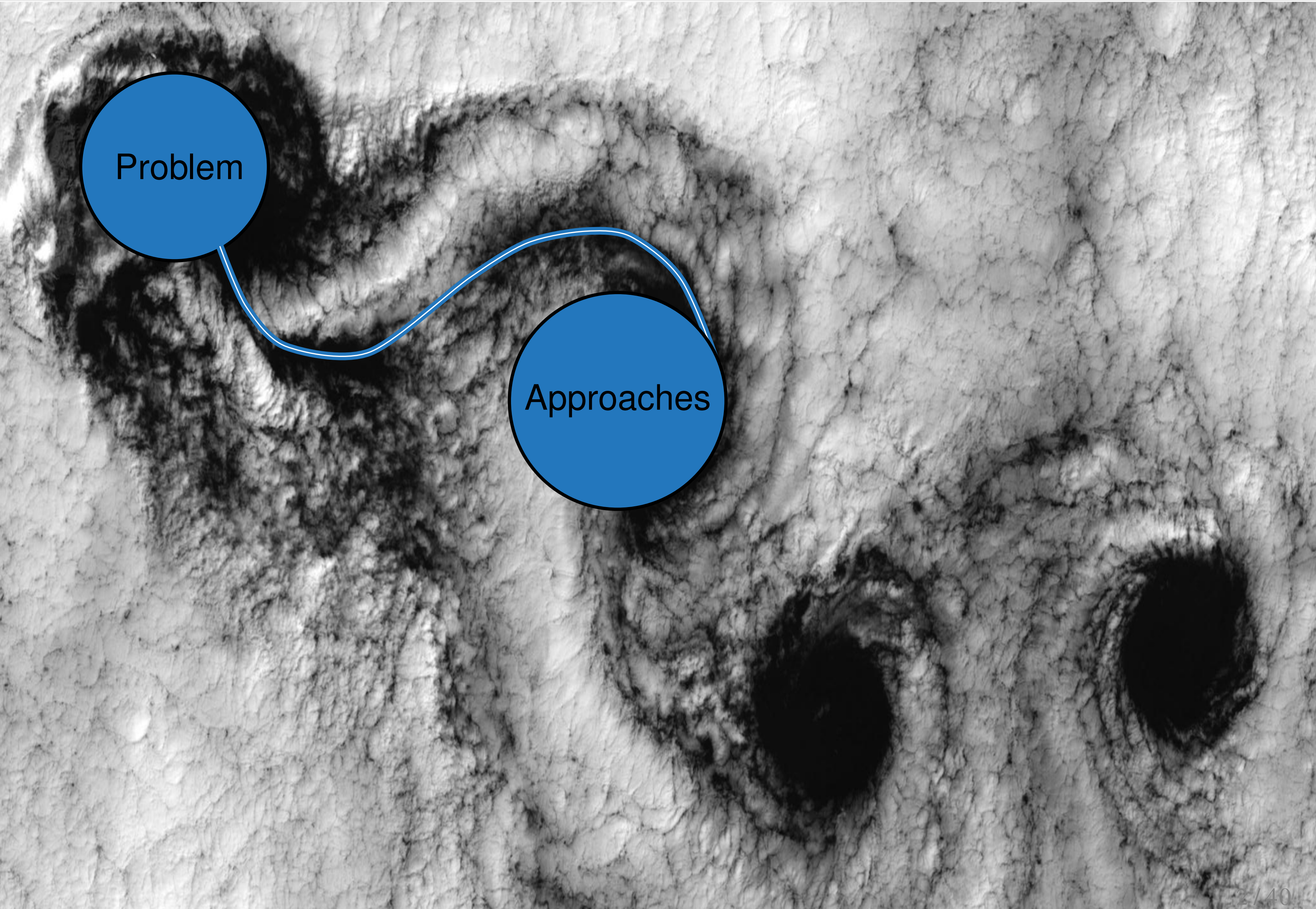
February 4, 2015

Flow-Informed Strategies

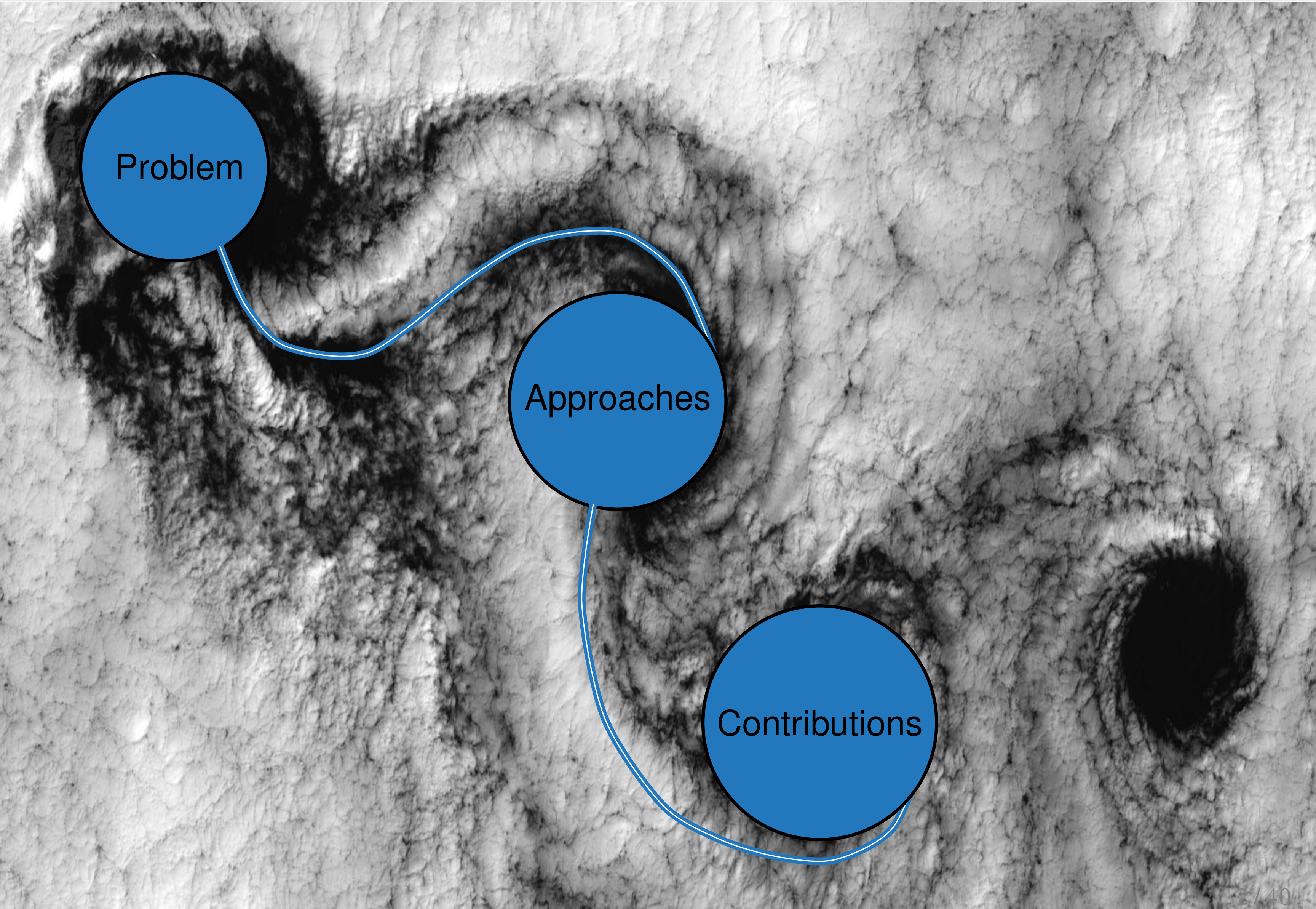


Flow-Informed Strategies

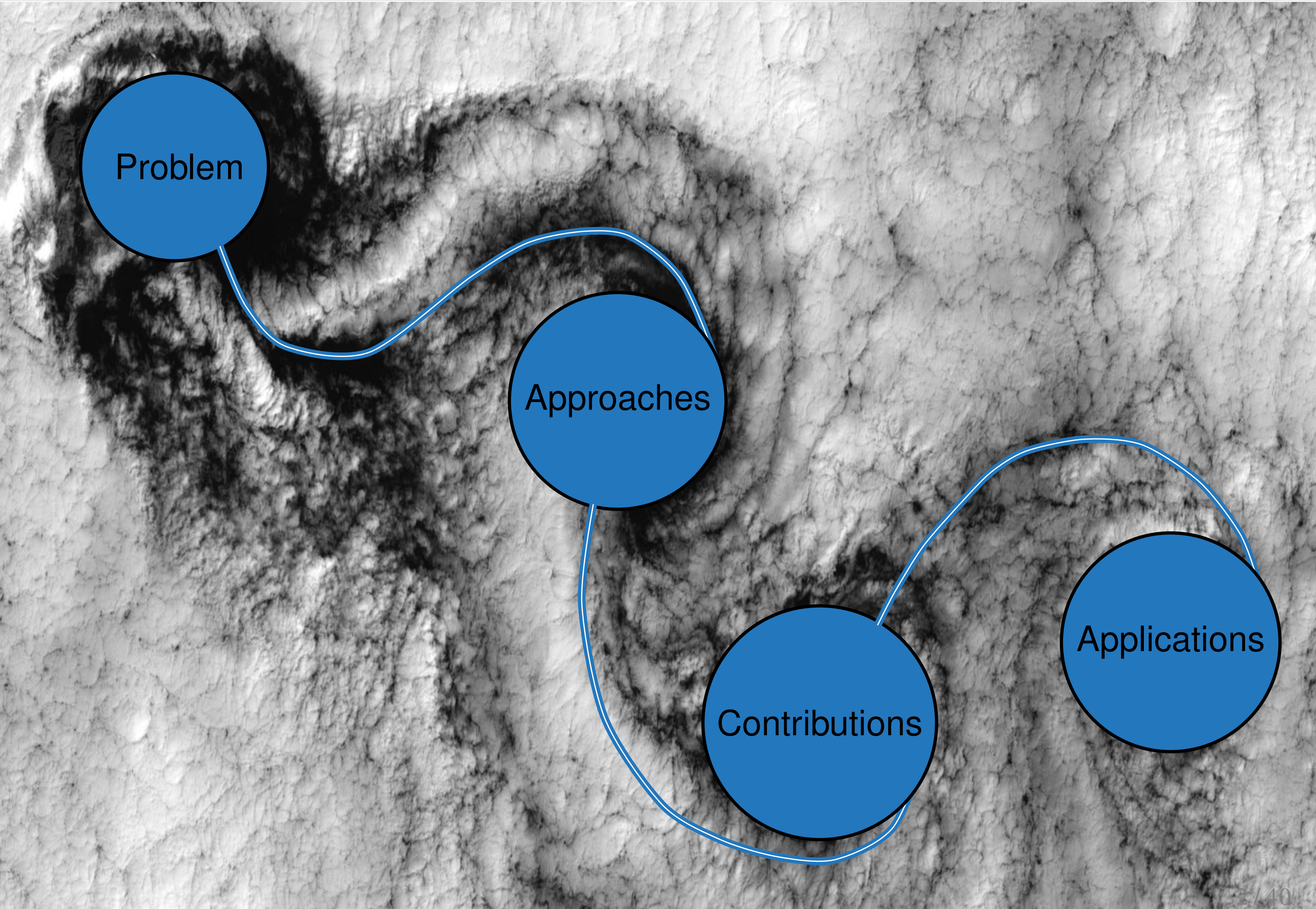



Problem
Approaches

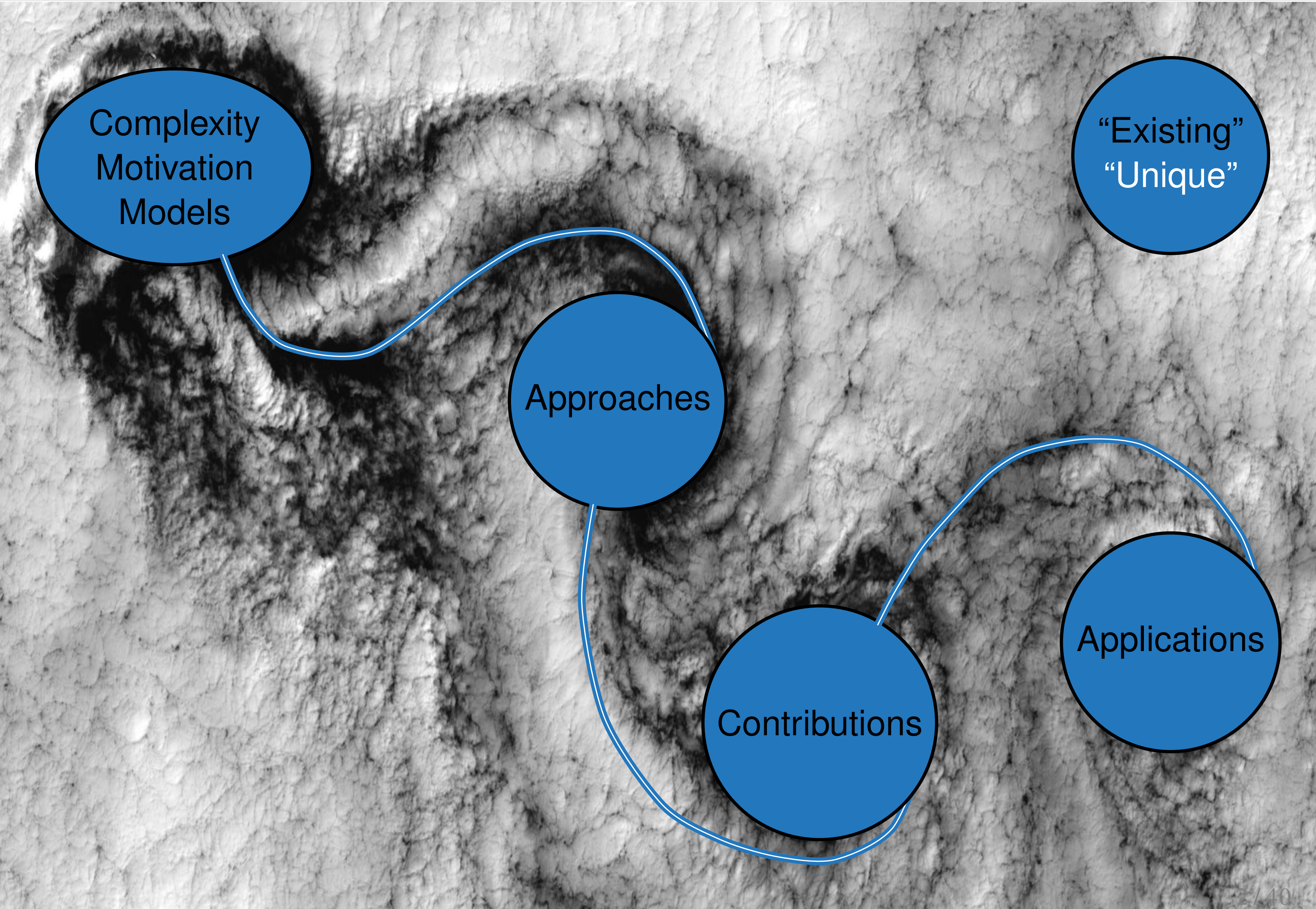
Flow-Informed Strategies



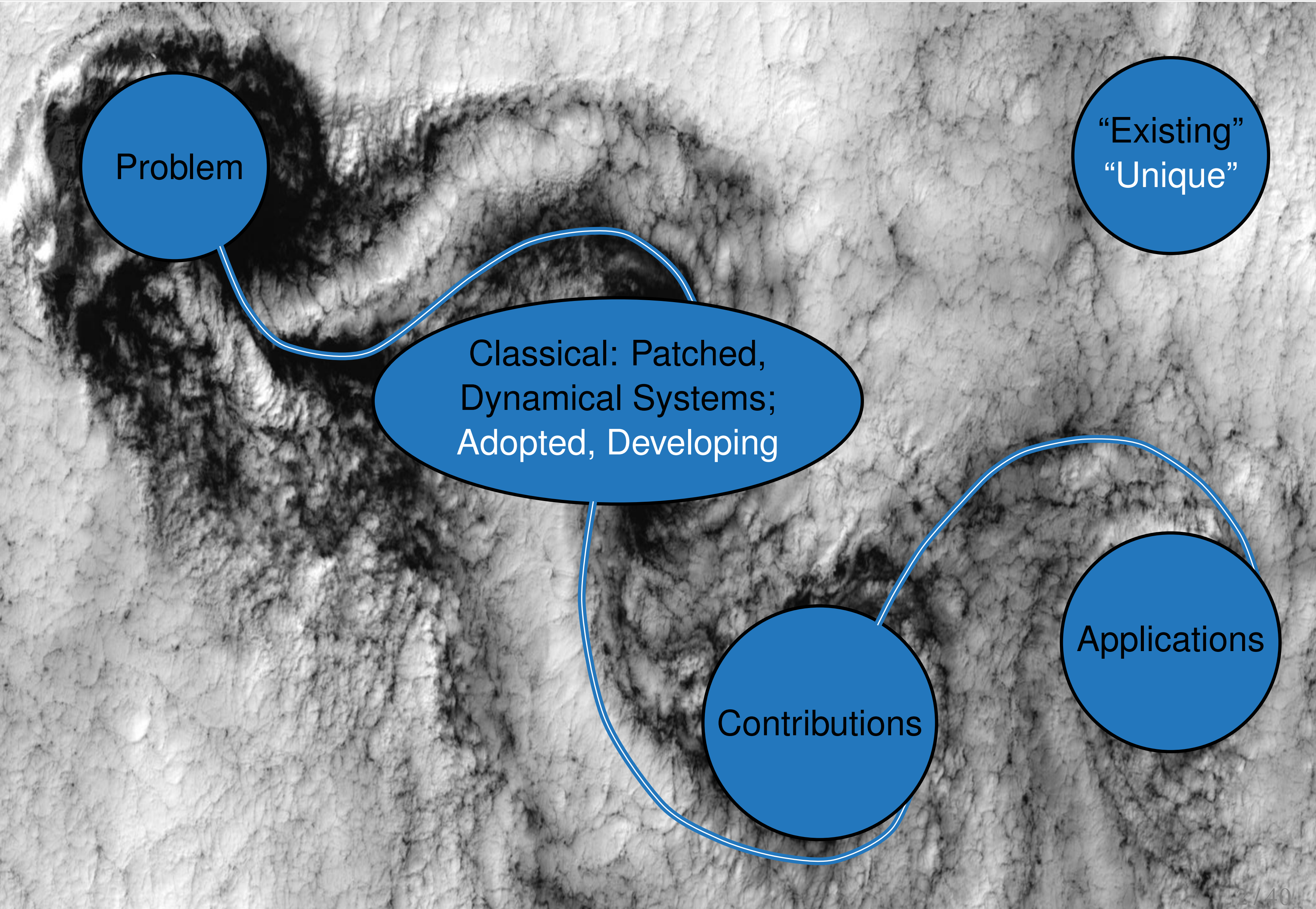
Flow-Informed Strategies



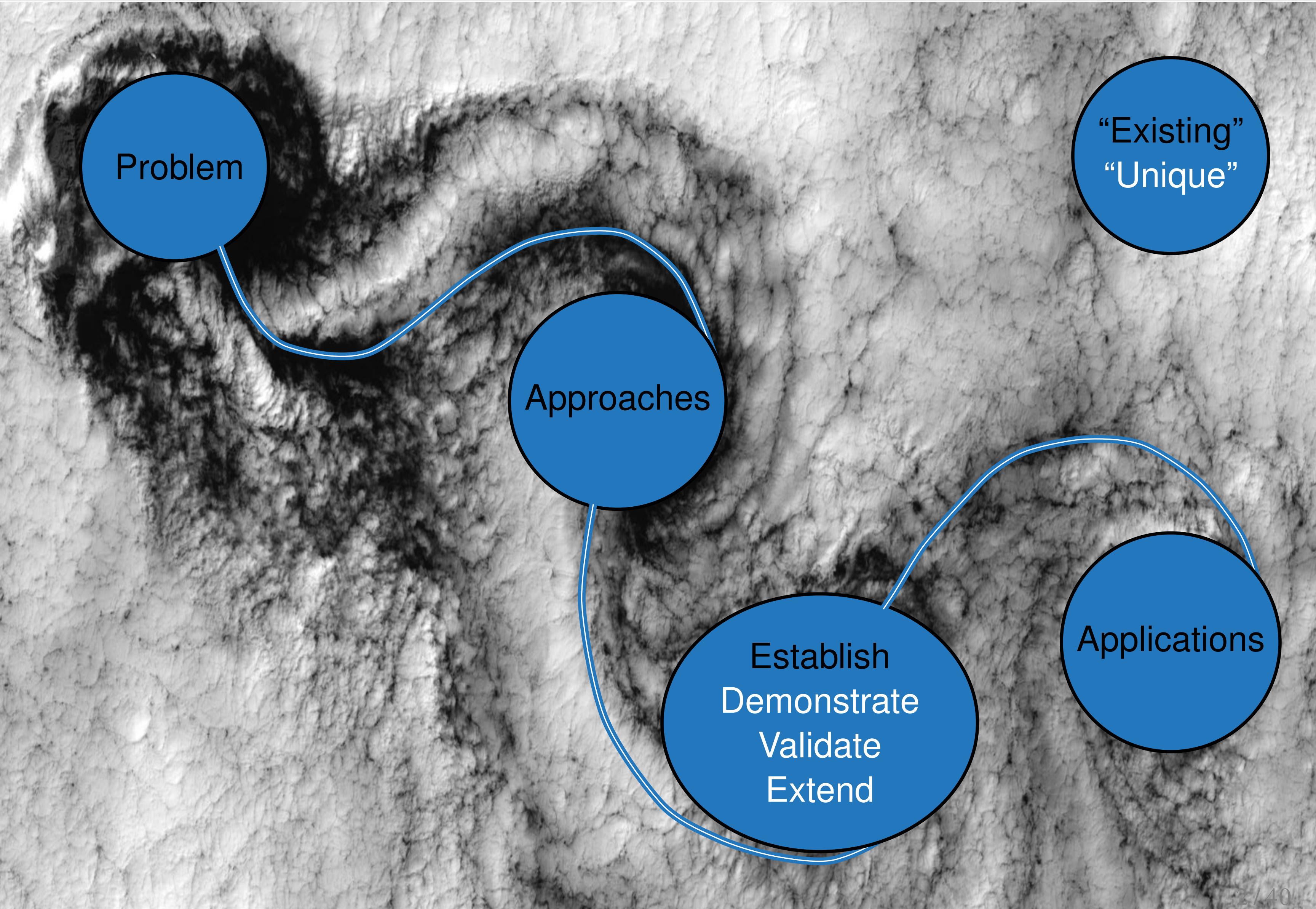
Flow-Informed Strategies

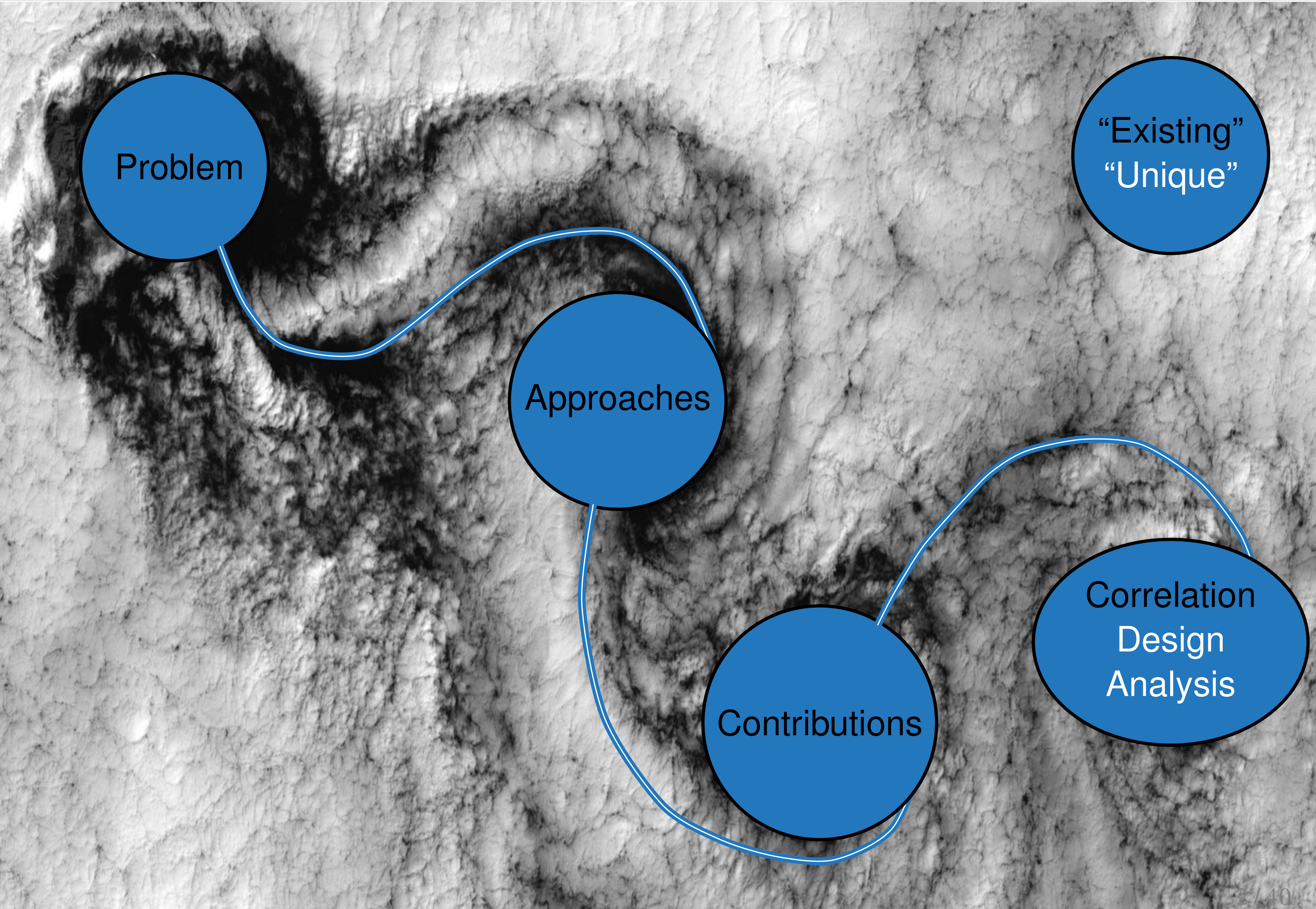


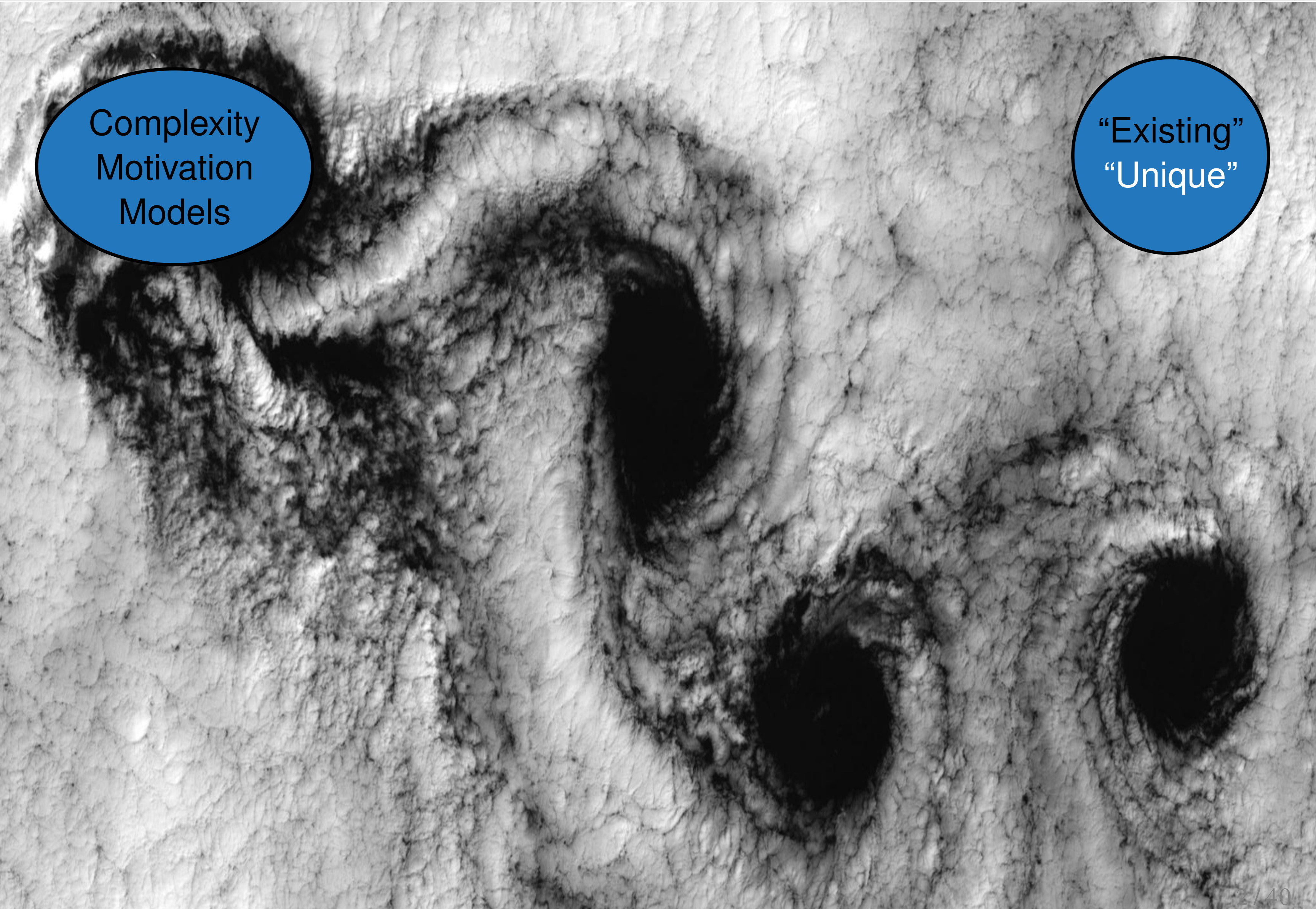
Flow-Informed Strategies



Flow-Informed Strategies




Problem
Approaches
Contributions
“Existing”
“Unique”
Correlation
Design
Analysis


Complexity
Motivation
Models
“Existing”
“Unique”

- Trajectory design and analysis frequently requires complex models incorporating many effects
- Working within increased-fidelity contexts reveals new insight
- Tools that apply across levels of model fidelity highlight the contributions of particular components
- Such methods also decrease the gap between preliminary design and mission-ready solutions...
- and identify many, potentially better, options

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Flow-based strategies extend existing methods and enable new methods

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Flow-based strategies extend existing methods and enable new methods

$\Rightarrow$ Goal: Apply and extend flow-based tools for spacecraft trajectory design in multi-body regimes

- Related flow-based concepts
  - Key theoretical development
    - Haller: 2000 (est.), 2001 (3D), 2011 (variational)
    - Haller and Beron-Vera: 2012 (geodesics)
    - Blazevski and Haller: 2013 (3D)
    - Teramoto et al.: 2013 (nD)
  - Astrodynamical applications
    - Gawlik et al.: 2009 (elliptic restricted problem)
    - Short et al.: 2011 (circular restricted problem)
    - Short and Howell: 2012 (higher-fidelity models)

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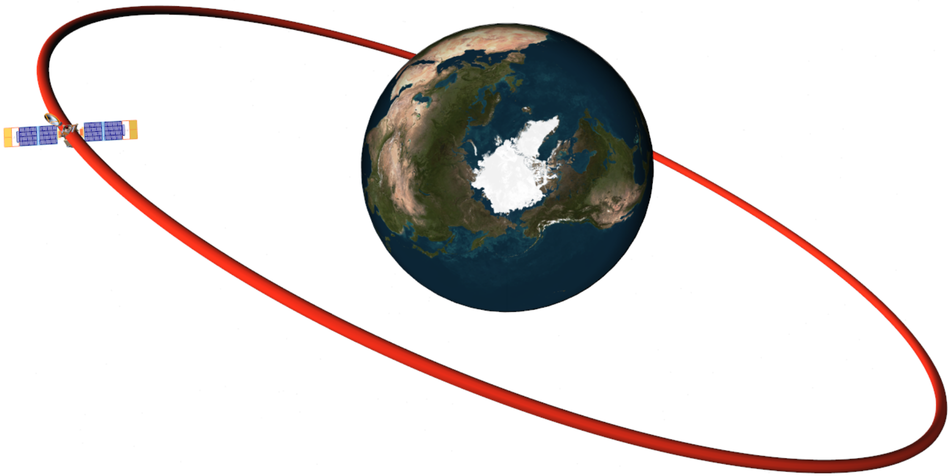
    - Short and Howell: 2012 (higher-fidelity models)
- Associated notions
  - Schroer & Ott: 1997 and Grebow: 2010 (control segments)
  - Anderson et al.: 2003 and Harden, et al.: 2014 (LLE)
  - Lara et al. 2007; Villac: 2008 and Villac and Broschart: 2009 (FLI)



Two-body Problem (2BP)

Flow-Informed Strategies

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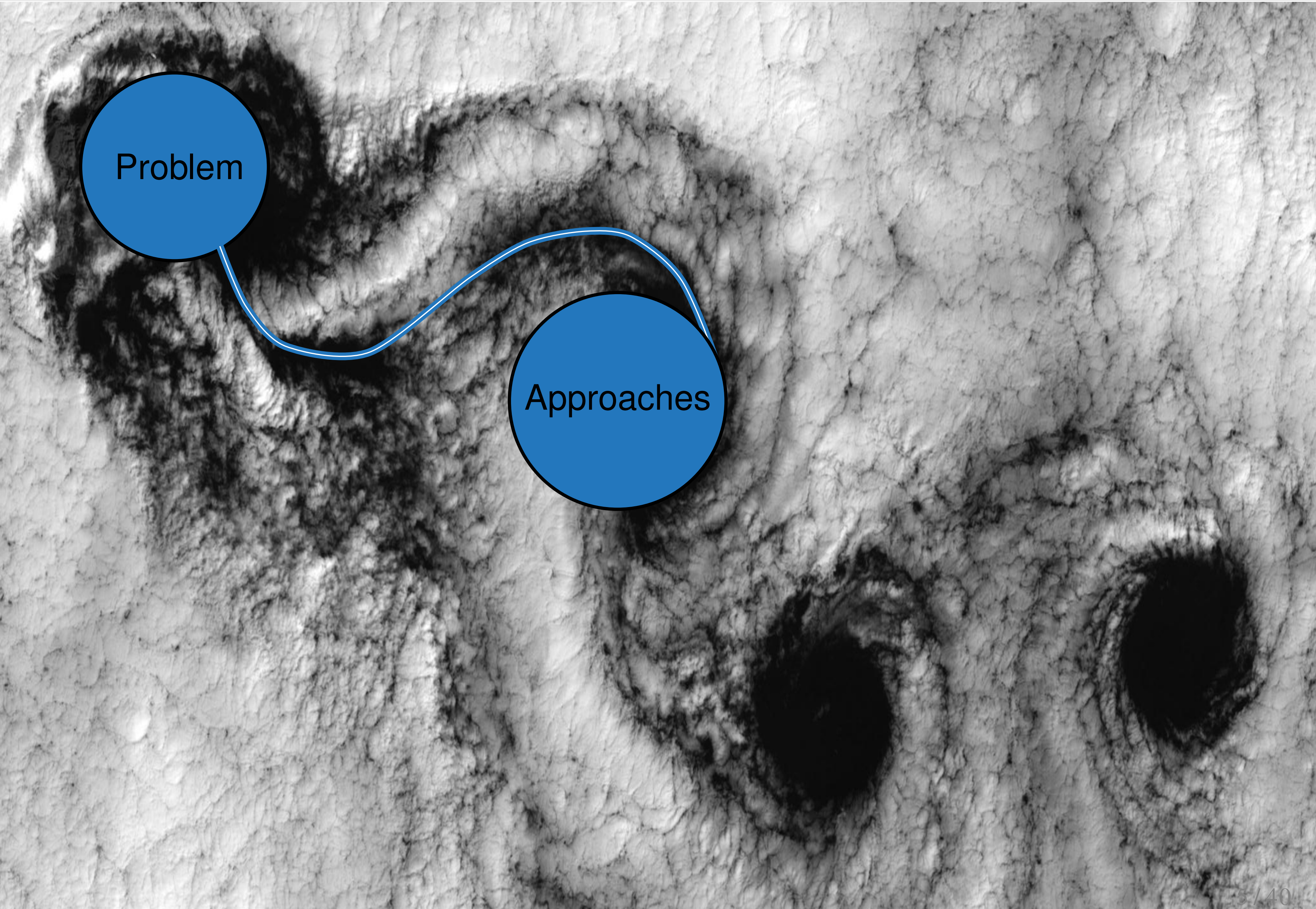
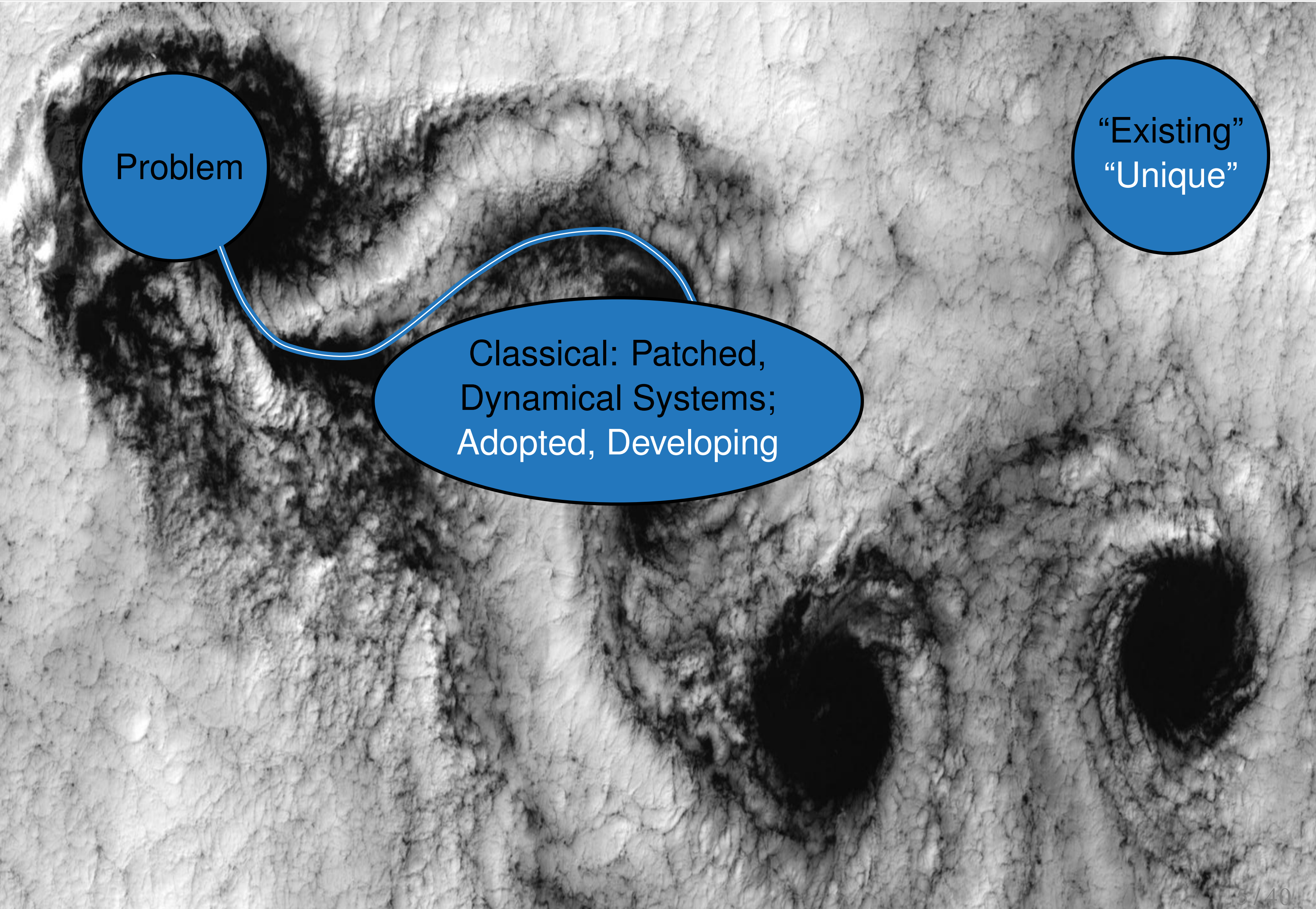


Flow-Informed Strategies



Classical Ideas

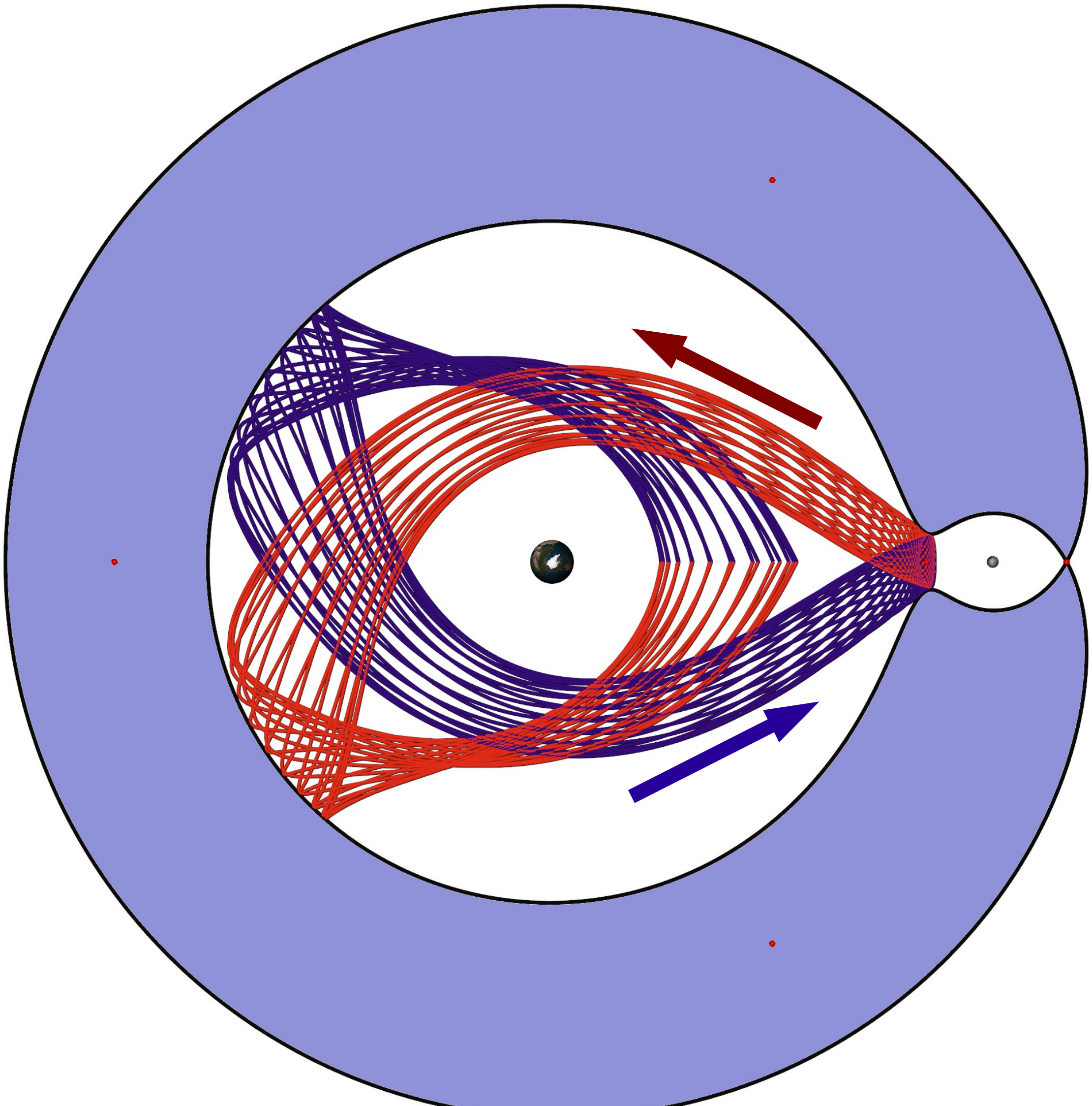
- Two-body conic strategies are well-established and frequently effective for particular situations
- Dynamical systems theory helps to define the underlying structure

Classical Ideas

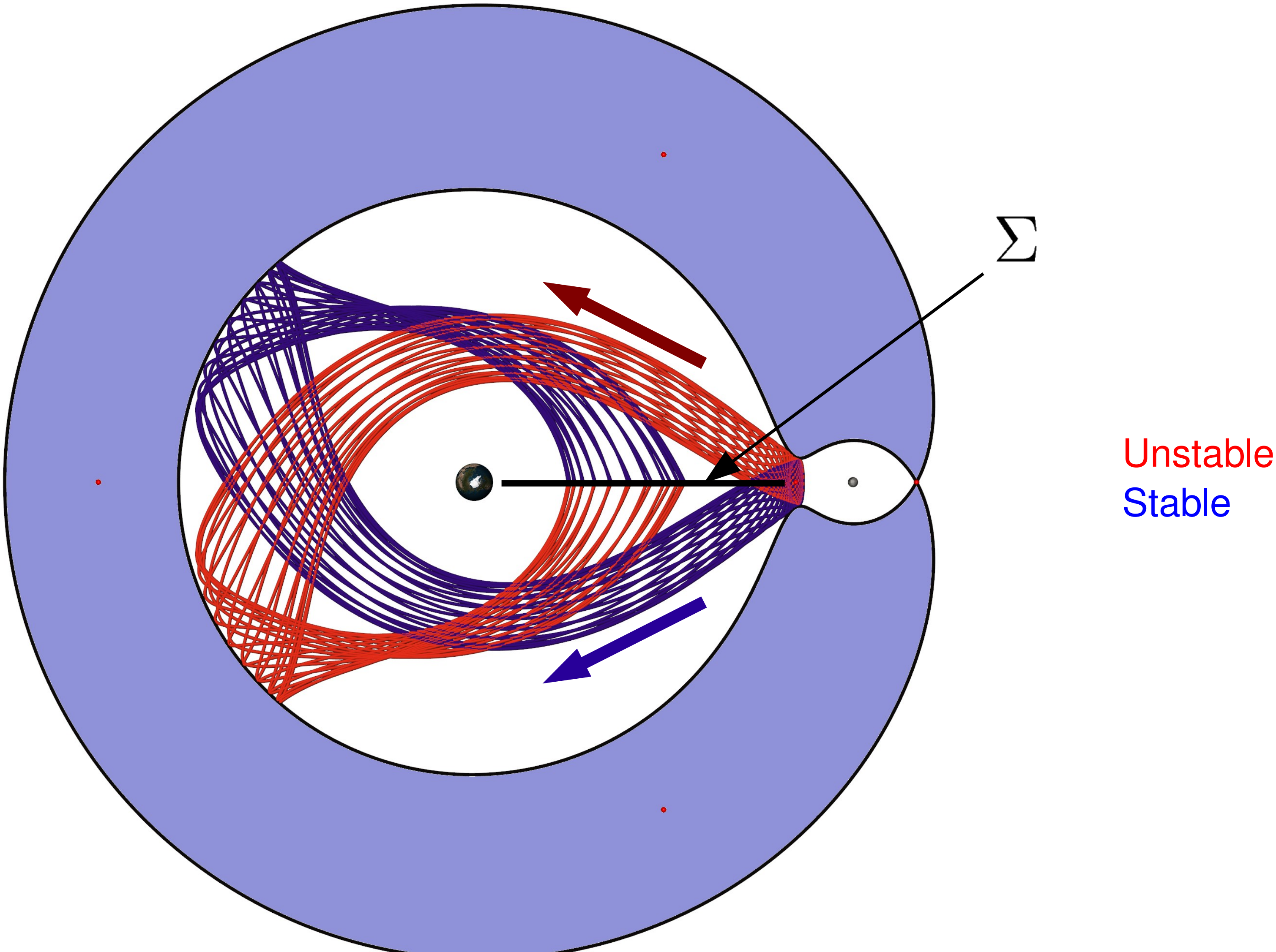
- Two-body conic strategies are well-established and frequently effective for particular situations
- Dynamical systems theory helps to define the underlying structure

Adopted and Developing Ideas

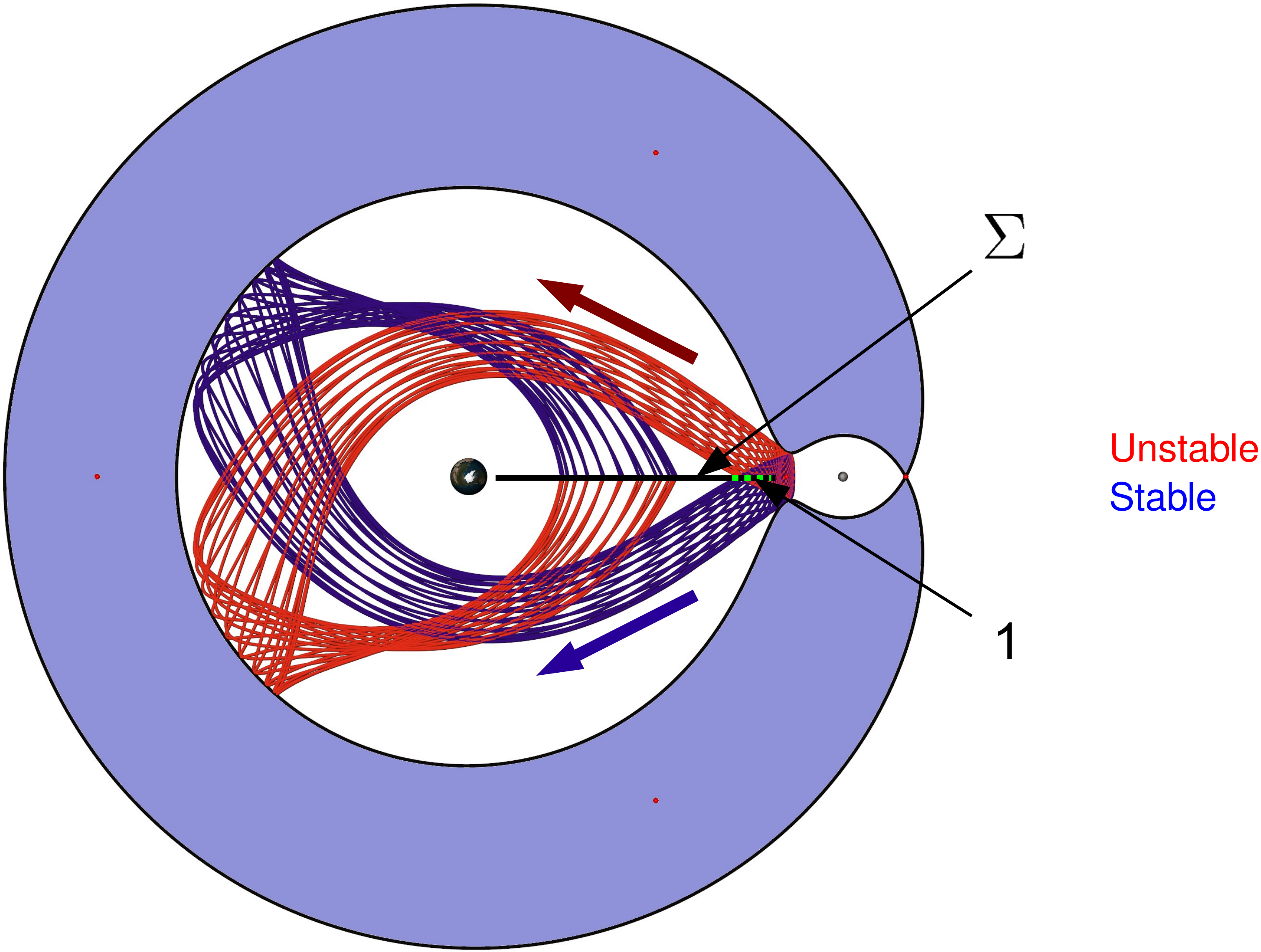
- Higher-fidelity models increase challenge in application of existing tools
- Direct modeling of the underlying flow reveals new options
- A possibility afforded by cumulation/extension of classical notions

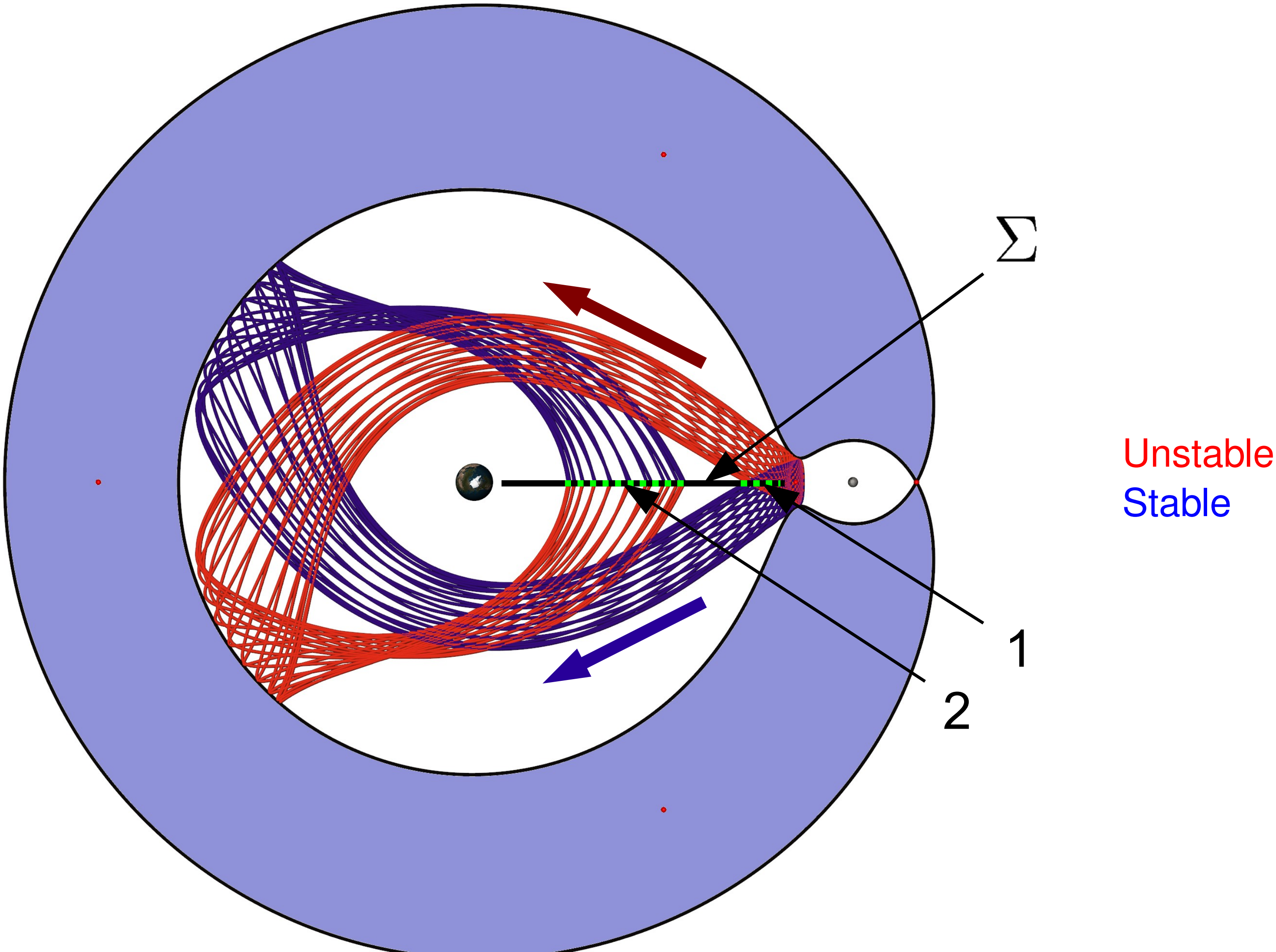


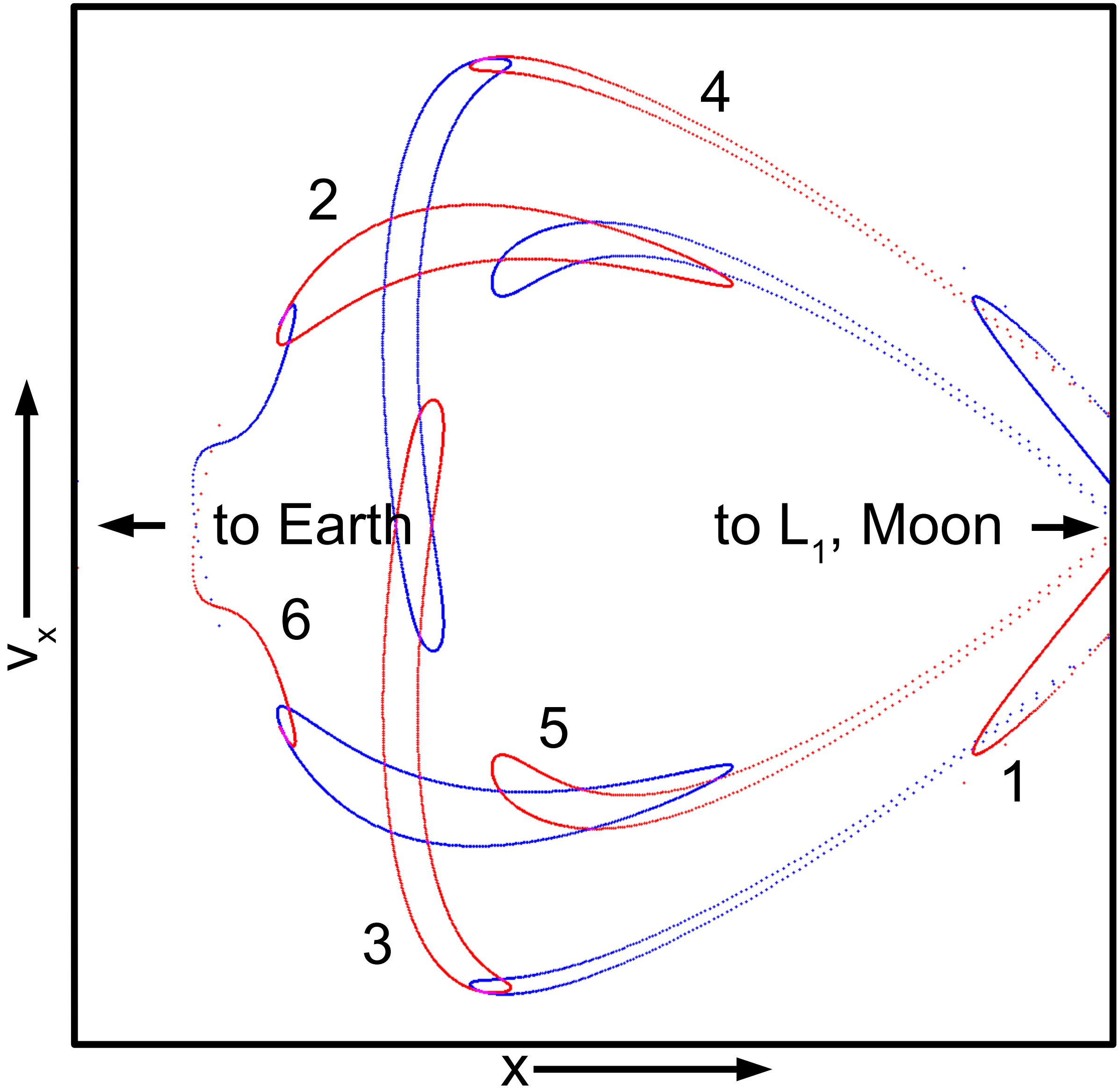
Unstable
Stable



Σ
Unstable
Stable

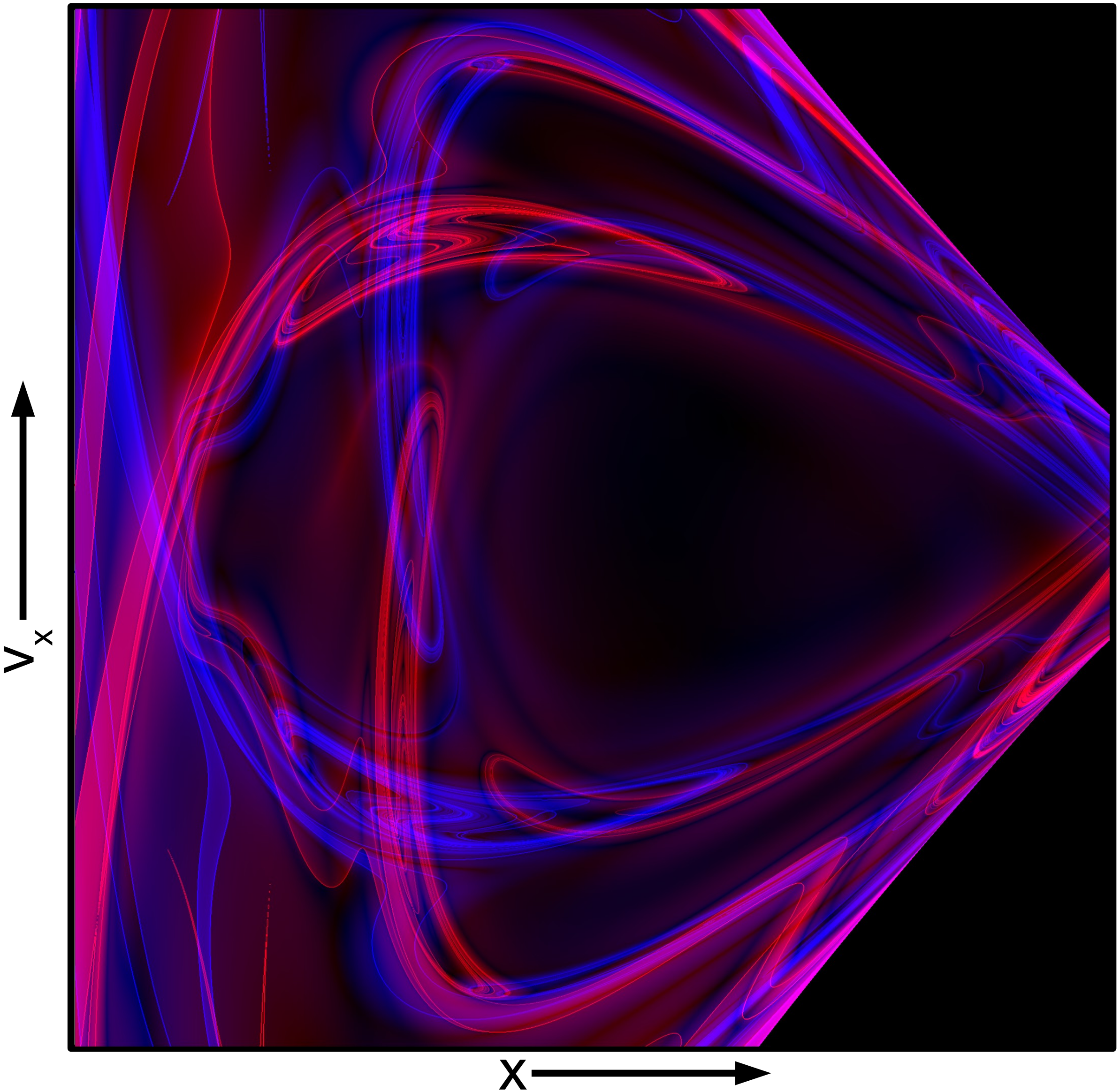

Σ
1
Unstable
Stable


Σ
1
2
Unstable
Stable



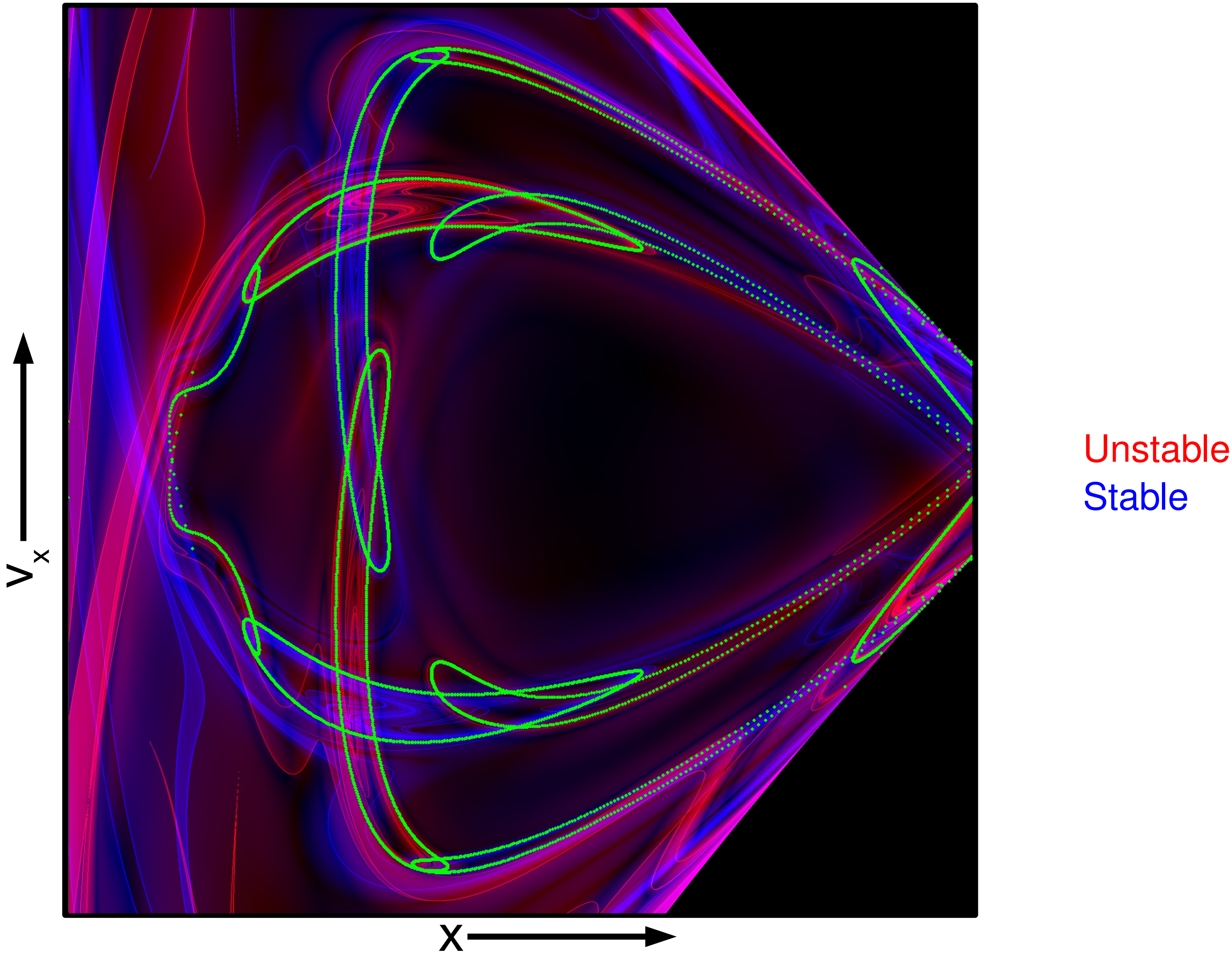
Unstable
Stable

FTLE Maps



Unstable

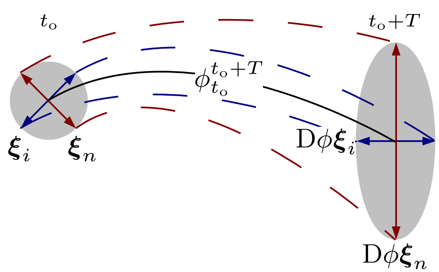
Stable


v_x
x
Unstable
Stable

- Flow map:

$$\mathbf{x}(t_o + T) = \phi_{t_o}^{t_o+T}(\mathbf{x}_{t_o})$$

- State transition matrix:



$$\Phi(t_o + T, t_o) = \frac{d\phi_{t_o}^{t_o+T}(\mathbf{x}_{t_o})}{d\mathbf{x}_{t_o}}$$

$$\delta\mathbf{x}(t_o + T) = \Phi(t_o + T, t_o)\delta\mathbf{x}(t_o)$$

- Cauchy-Green Strain Tensor:

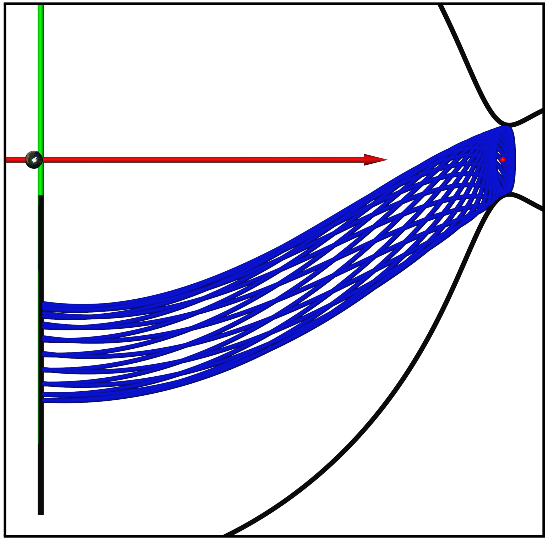
$$\Phi^{\mathrm{T}}\Phi$$

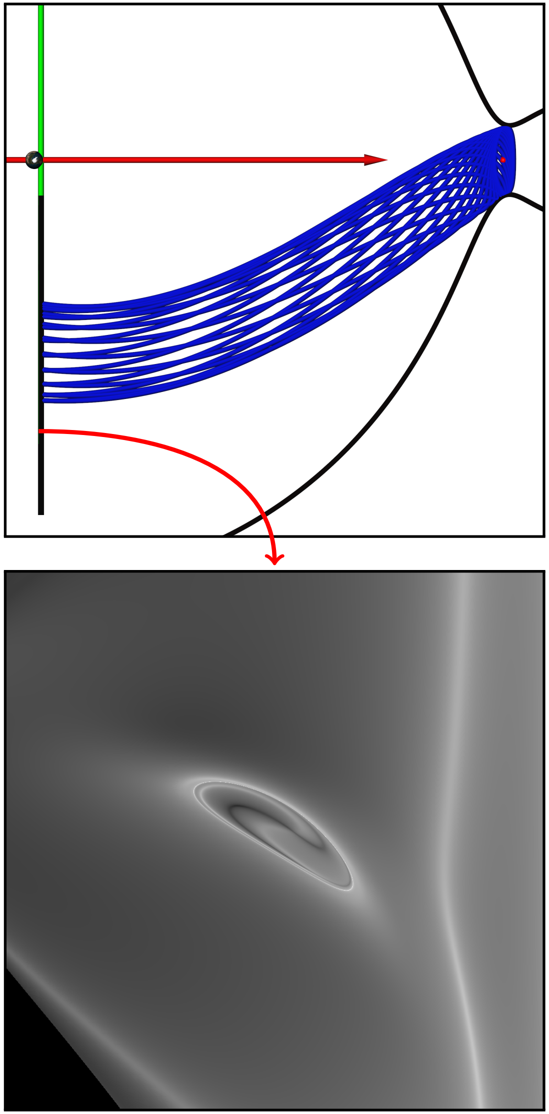
Stretching in the phase space along eigenvector $\boldsymbol{\xi}_i \propto \sqrt{\lambda_i}$

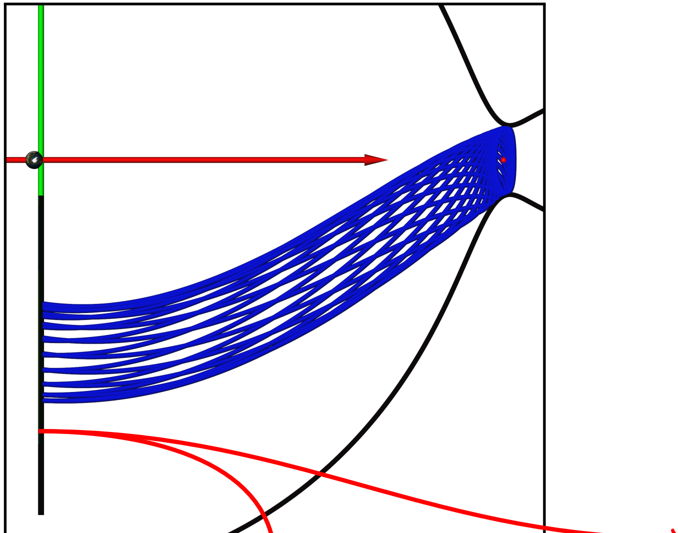
- Finite-time Lyapunov exponents:

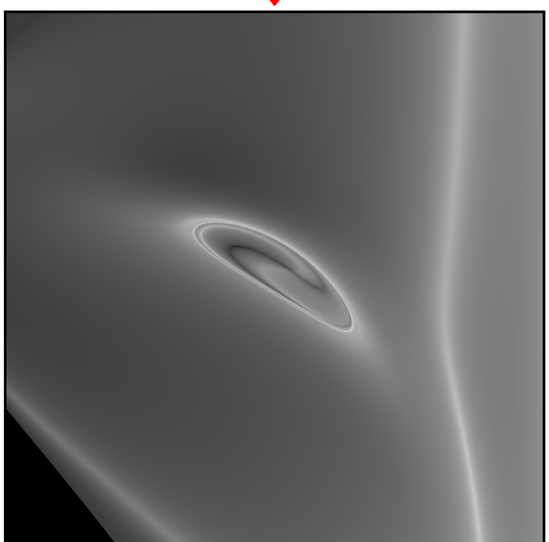
$$\Lambda_i = \frac{1}{|T|} \ln \sqrt{\lambda_i\left(\Phi^{\mathrm{T}}\Phi\right)}$$

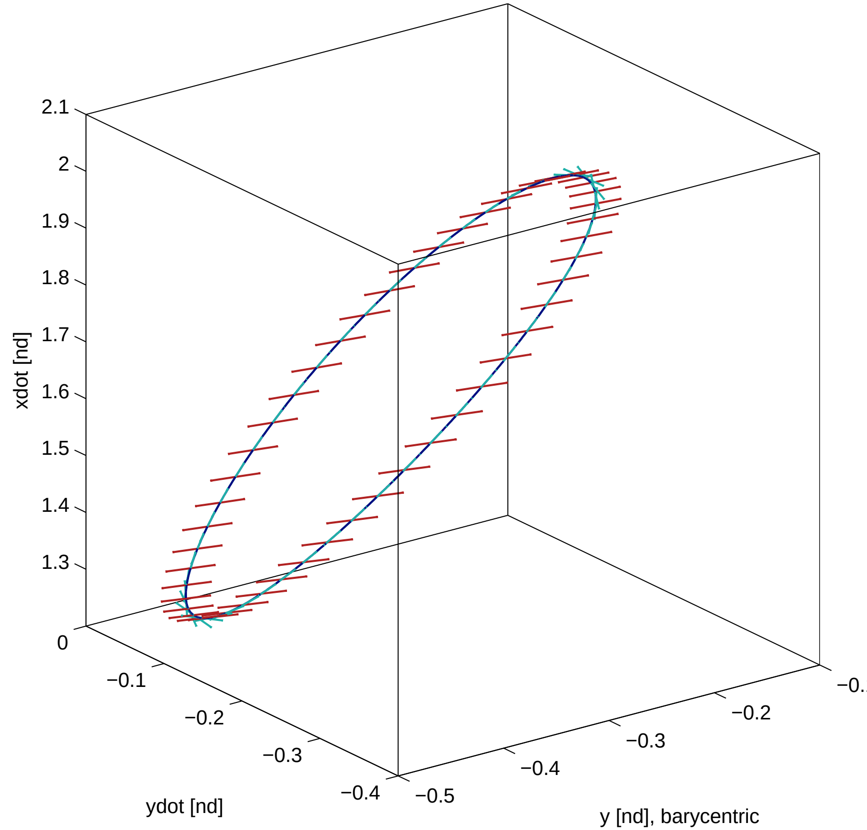
Supply a relative measure of stretching for trajectories
Generally interested in Λ_{max}







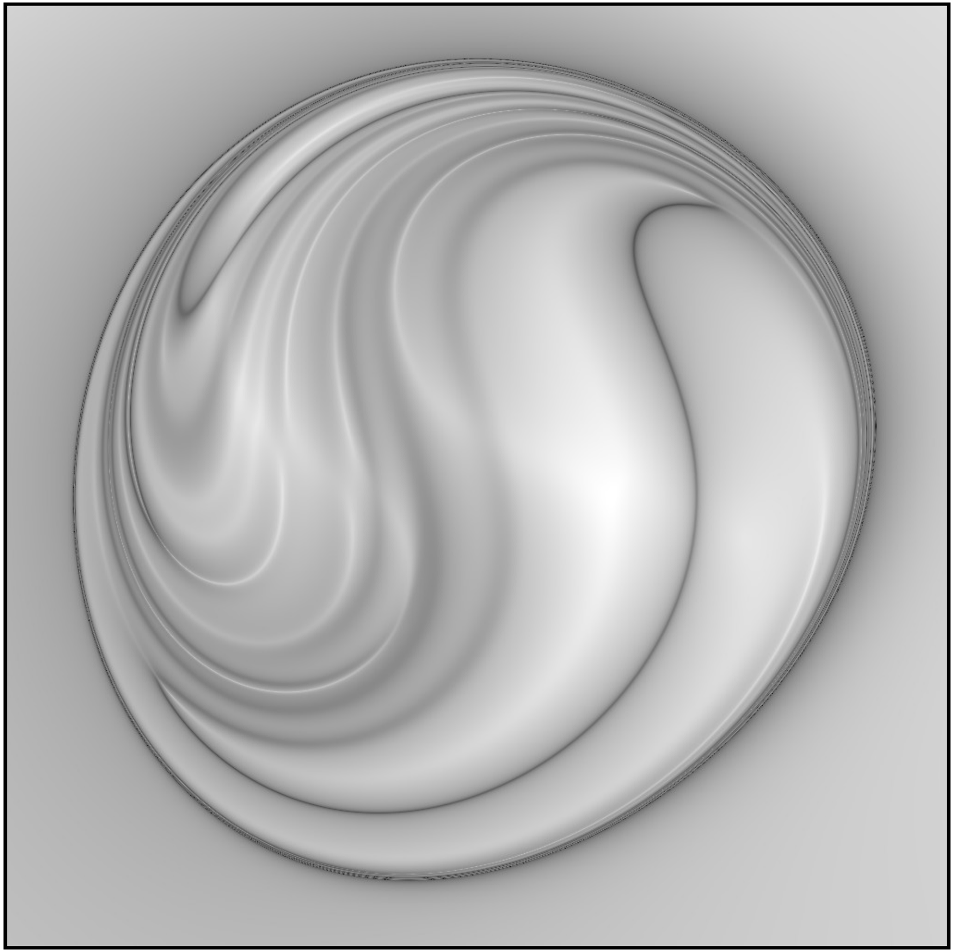



2.1
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
0
xdot [nd]
−0.1
−0.2
−0.3
−0.4
ydot [nd]
−0.5
−0.4
−0.3
−0.2
−0.1
y [nd], barycentric

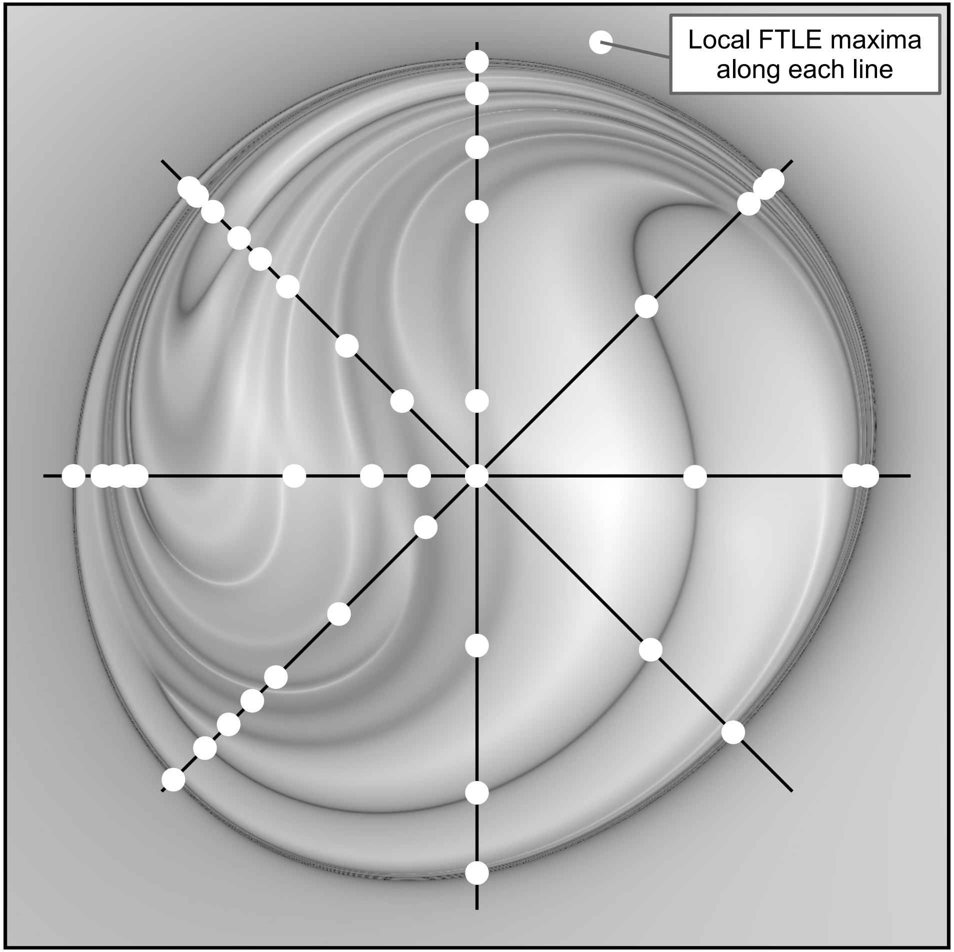
FTLE map features $\Longleftrightarrow$ LCS $\Longleftrightarrow$ Flow-governing structures

- Each notion emerges from the same fundamental mathematics
- Should be able to reliably identify structures from map features
- Numerically, algorithmically challenging
- One approach highlighted

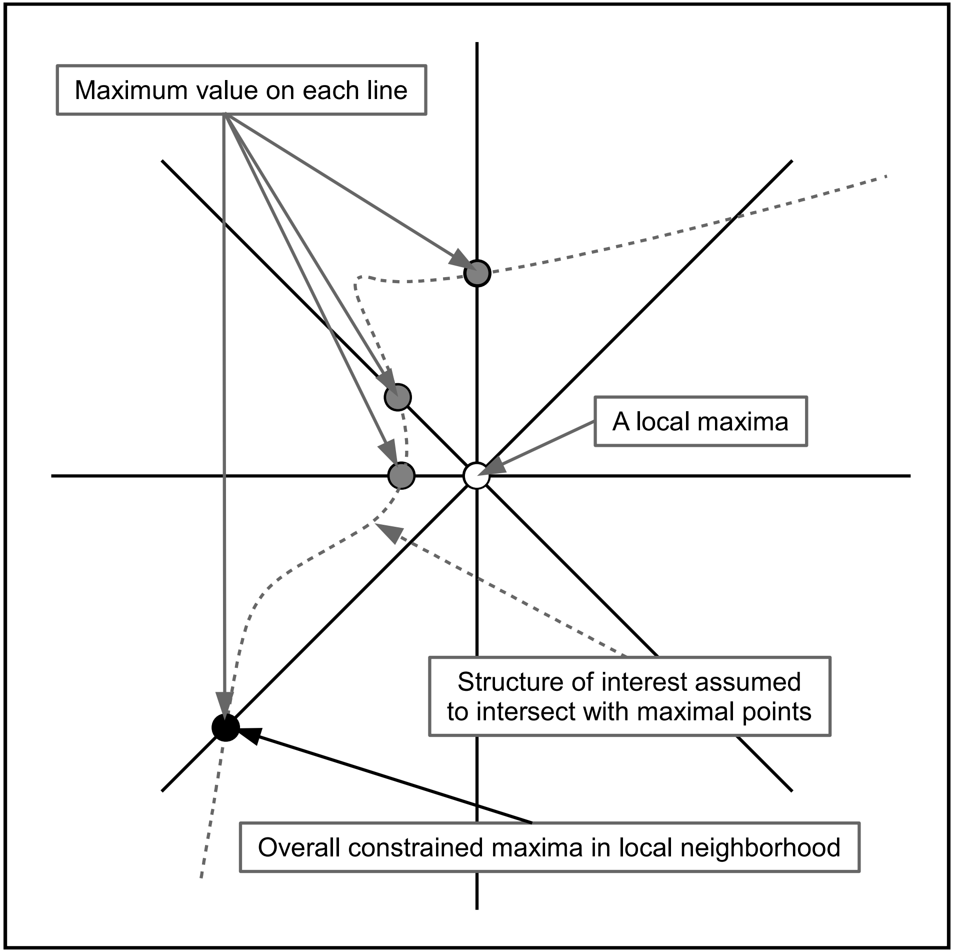
LCS Candidate Points



FTLE value: small, large



FTLE value: small, large


Maximum value on each line
A local maxima
Structure of interest assumed
to intersect with maximal points
Overall constrained maxima in local neighborhood

FTLE map features $\Longleftrightarrow$ LCS $\Longleftrightarrow$ Flow-governing structures

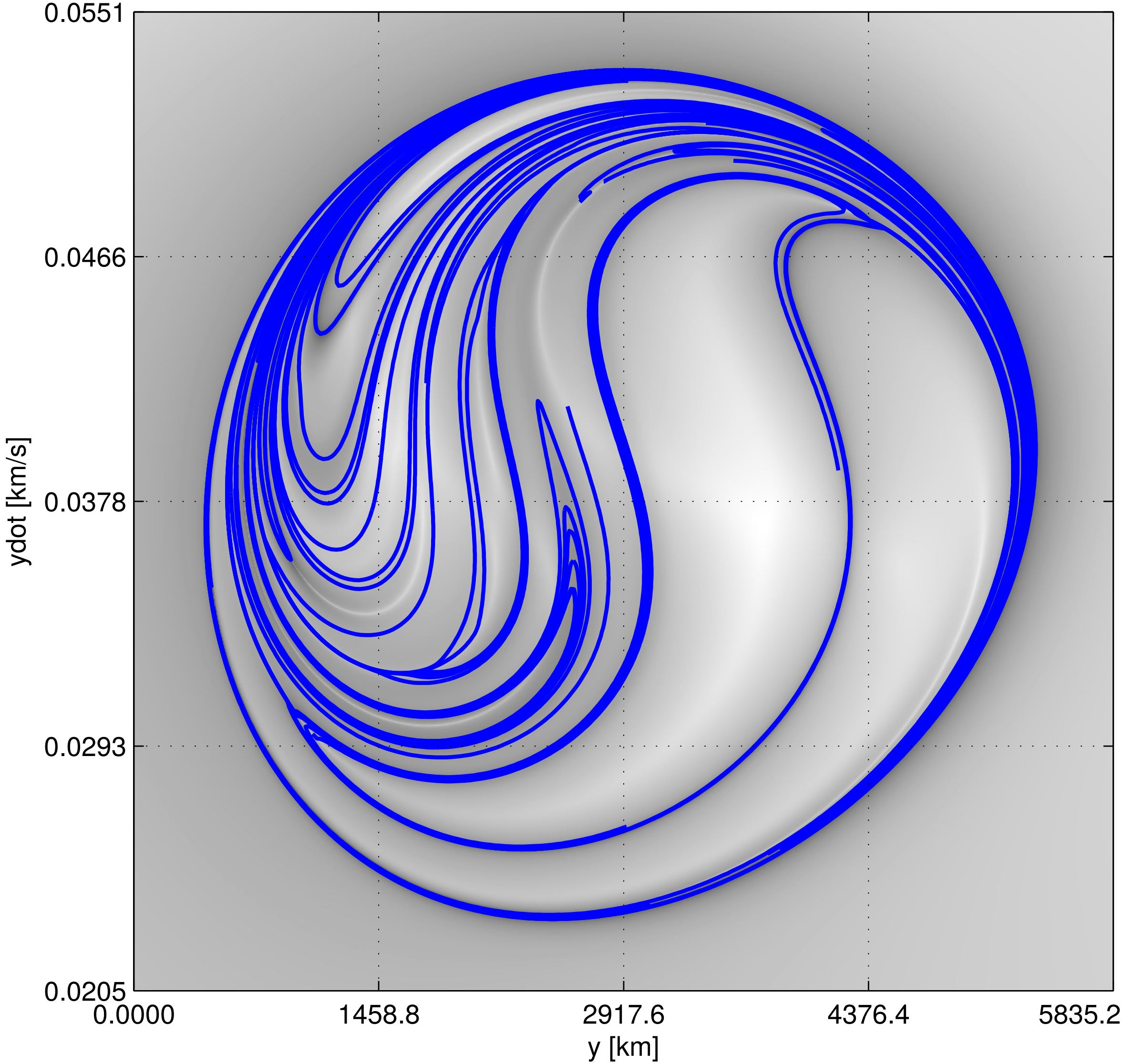
- Each notion emerges from the same fundamental mathematics
- Should be able to reliably identify structures from map features
- Numerically, algorithmically challenging
- One approach highlighted
- Given sufficiently refined candidate points
  $\Rightarrow$ eigenvectors and other associated vectors trace structures

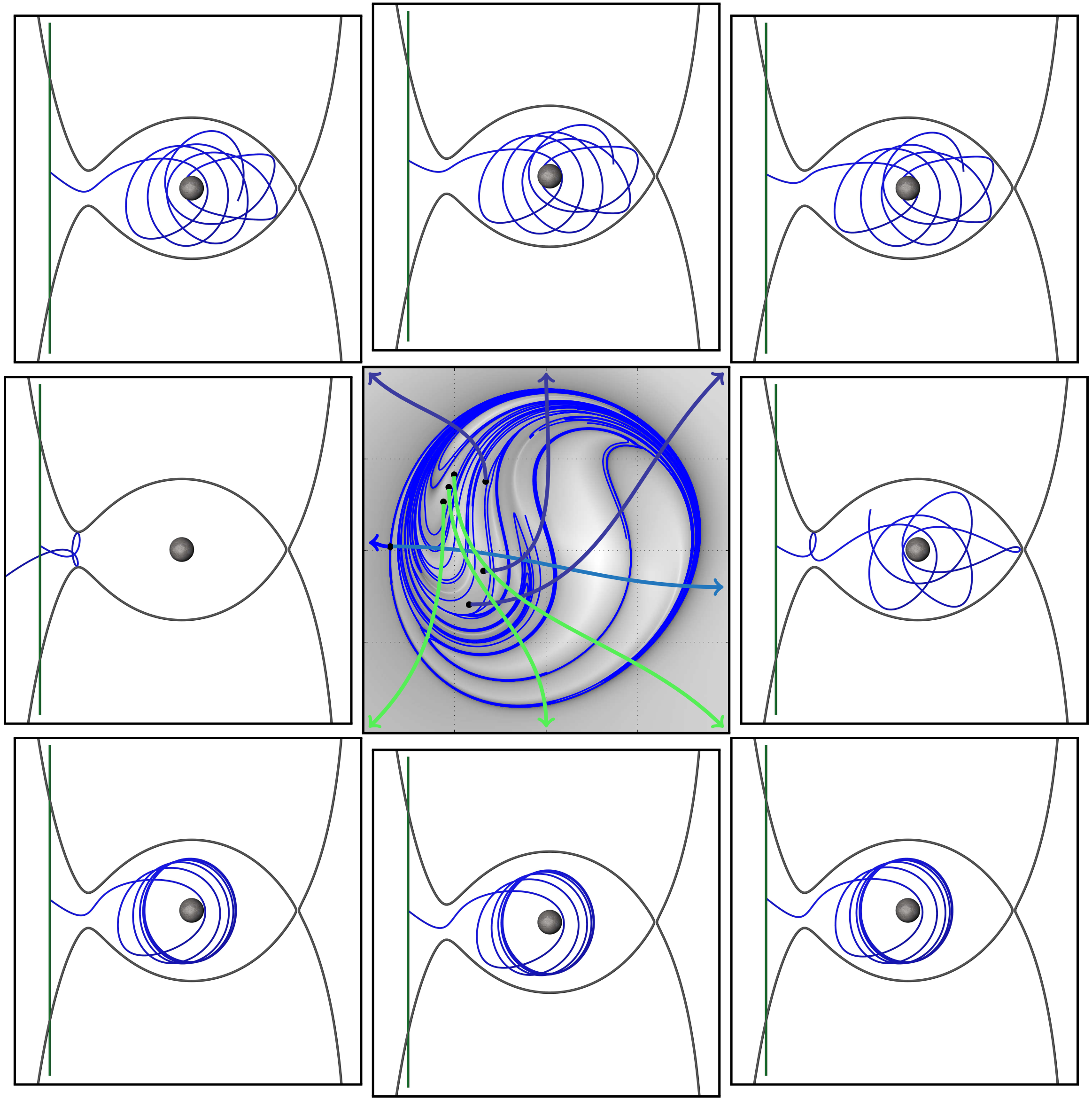
- LCS are orthogonal to ξ_n
- In higher-dimensional flows additional constraints required
- Example normals selected: $\boldsymbol{n}_1 = \nabla C$ and $\boldsymbol{n}_2 = (1, 0, 0, 0)$
- Structure tangents, $\hat{\xi}_n$, are mutually orthogonal to $\boldsymbol{n}_1$, $\boldsymbol{n}_2$, and ξ_n
- The associated curves, *strainlines*, are numerically integrated through the tangent-vector field for parametrizations of $\mathbf{r}$:

$$\frac{d}{ds}\mathbf{r}(s) = \operatorname{sign}\left(\left\langle \hat{\xi}_n(\mathbf{r}(s-\Delta)), \hat{\xi}_n(\mathbf{r}(s))\right\rangle\right) \hat{\xi}_n(\mathbf{r}(s))$$

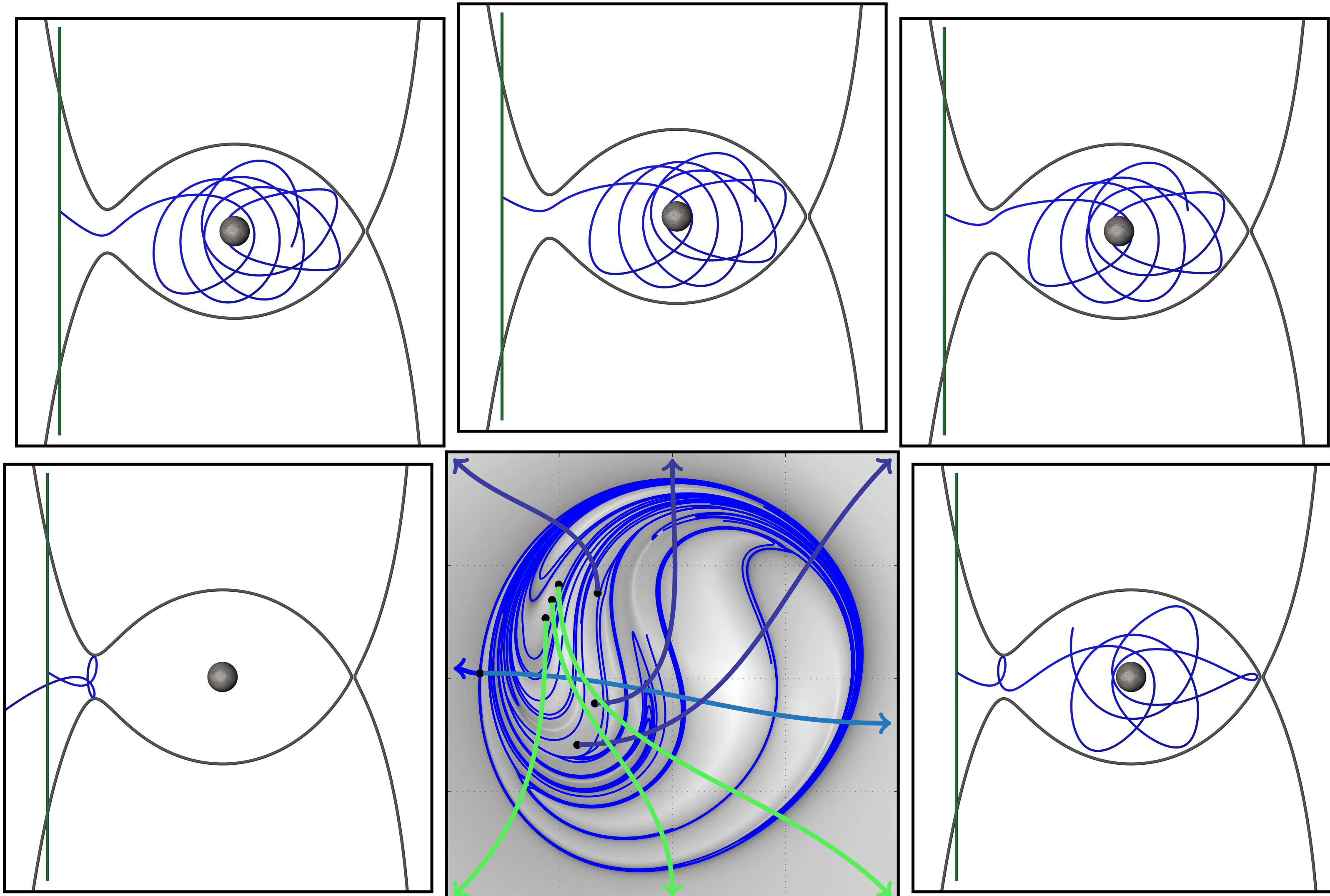
- As $\hat{\xi}_n$ inherits lack of sense from ξ_n, the previous step is considered

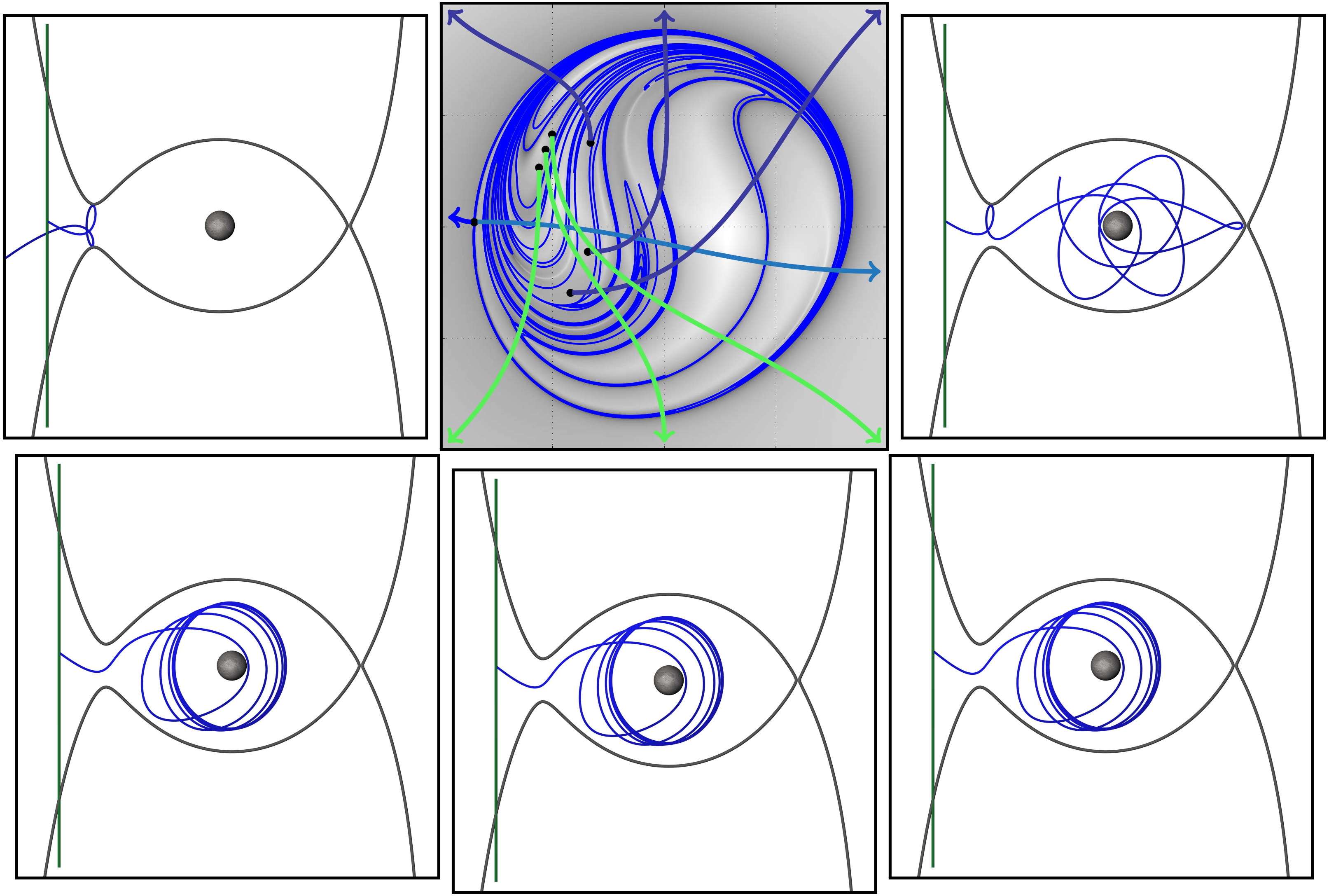
Resulting Strainline Map Projection

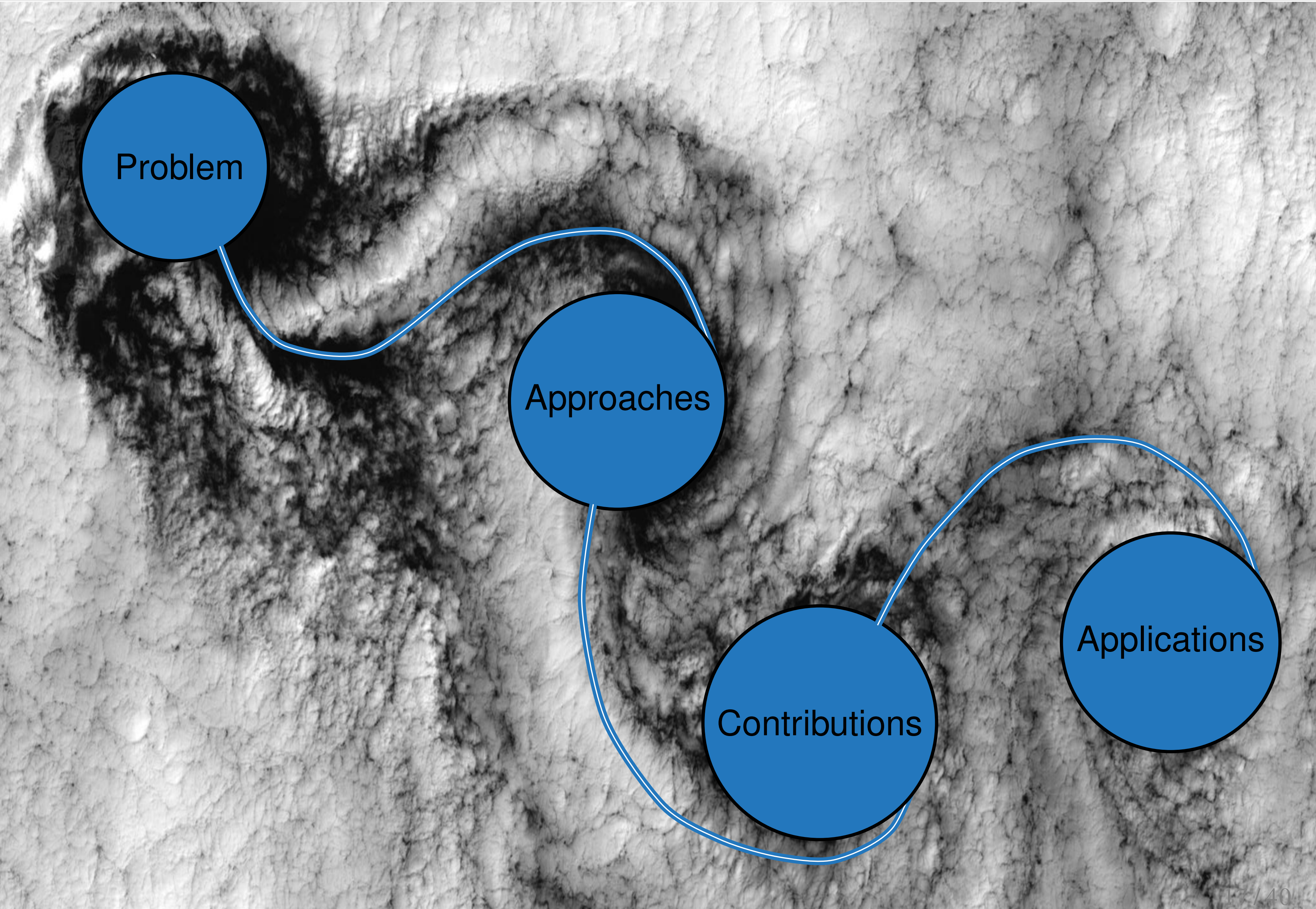




Strainline Region Isolation






Problem
Approaches
Contributions
Applications

Flow-Informed Strategies

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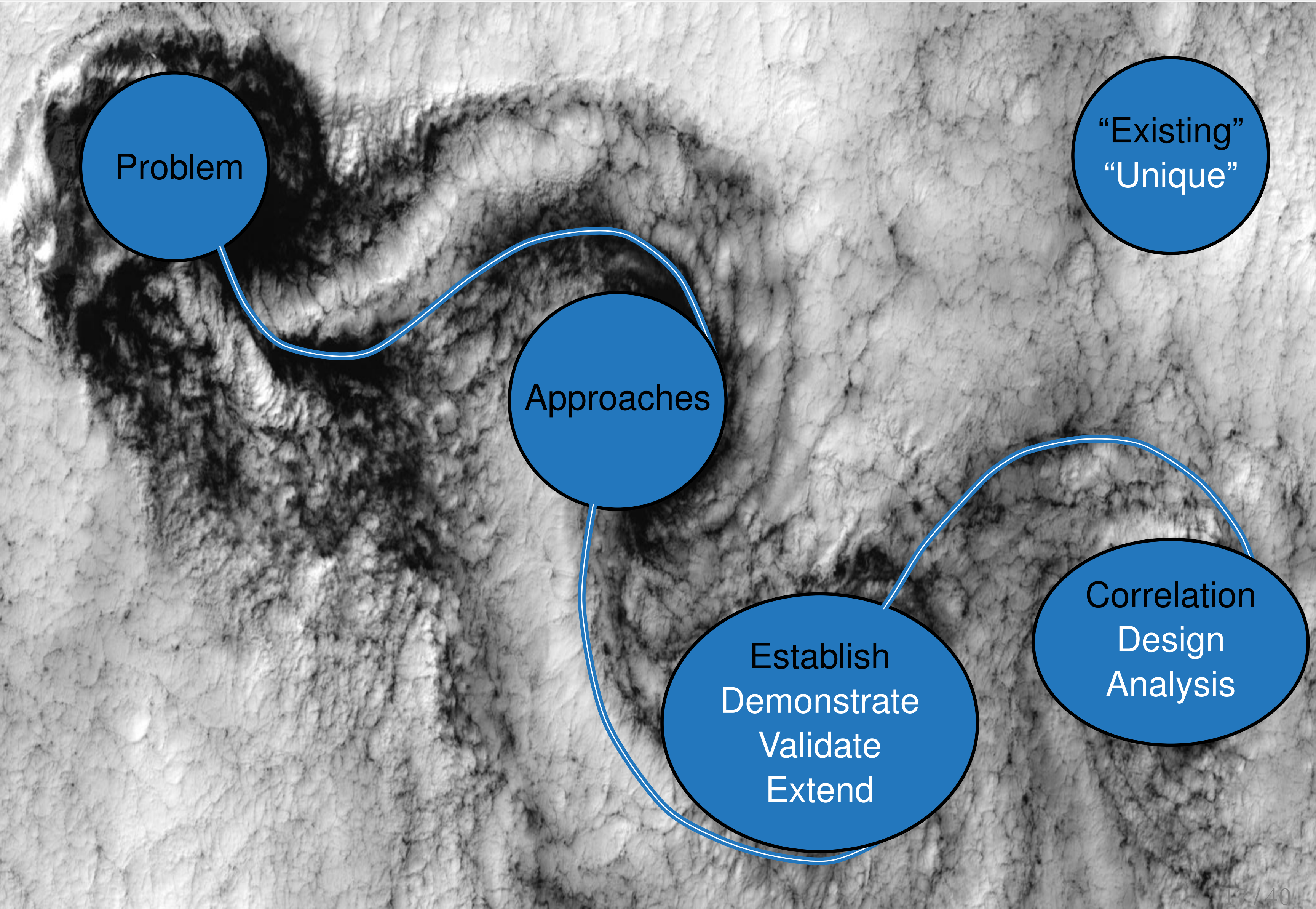

- Demonstrate and Establish
  - FTLE/LCS correlation
  - Periapse maps and the FILE
- Validate
  - Model fidelity impact on flow structures
  - Maneuver direction analysis
- Develop and Extend
  - Flow control segments
  - Trajectory design example in a nonautonomous system

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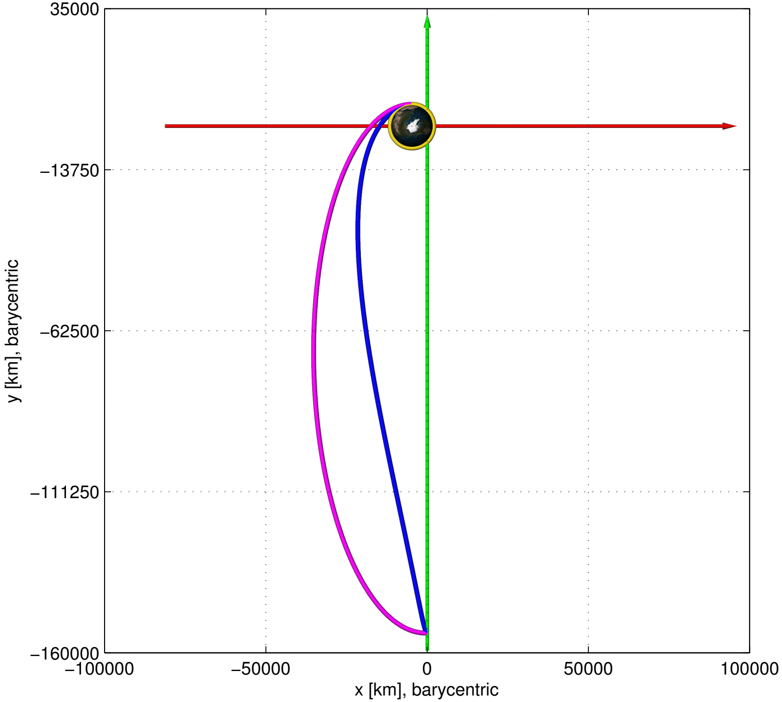
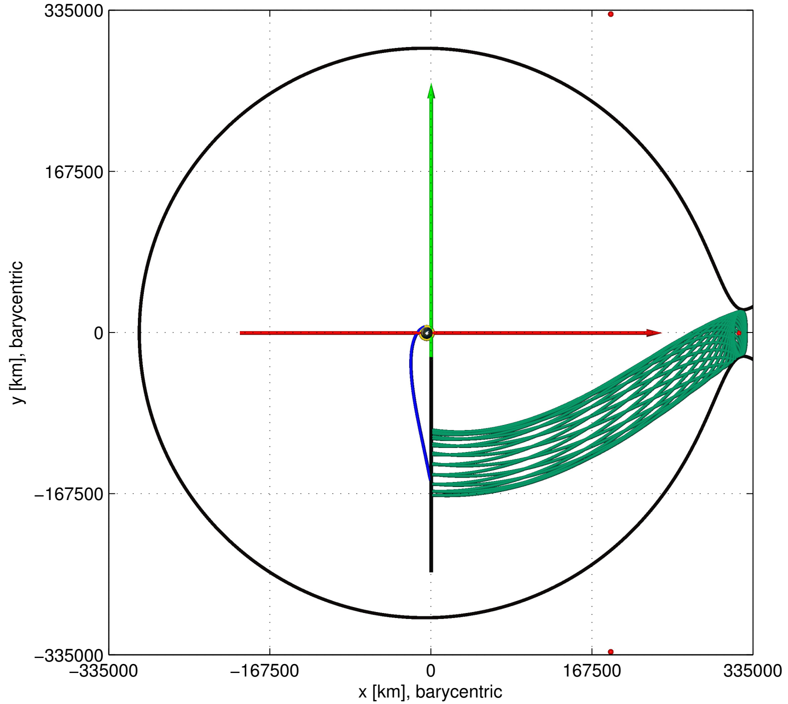
Hohmann arc:
problem framing and model
transitioning notion

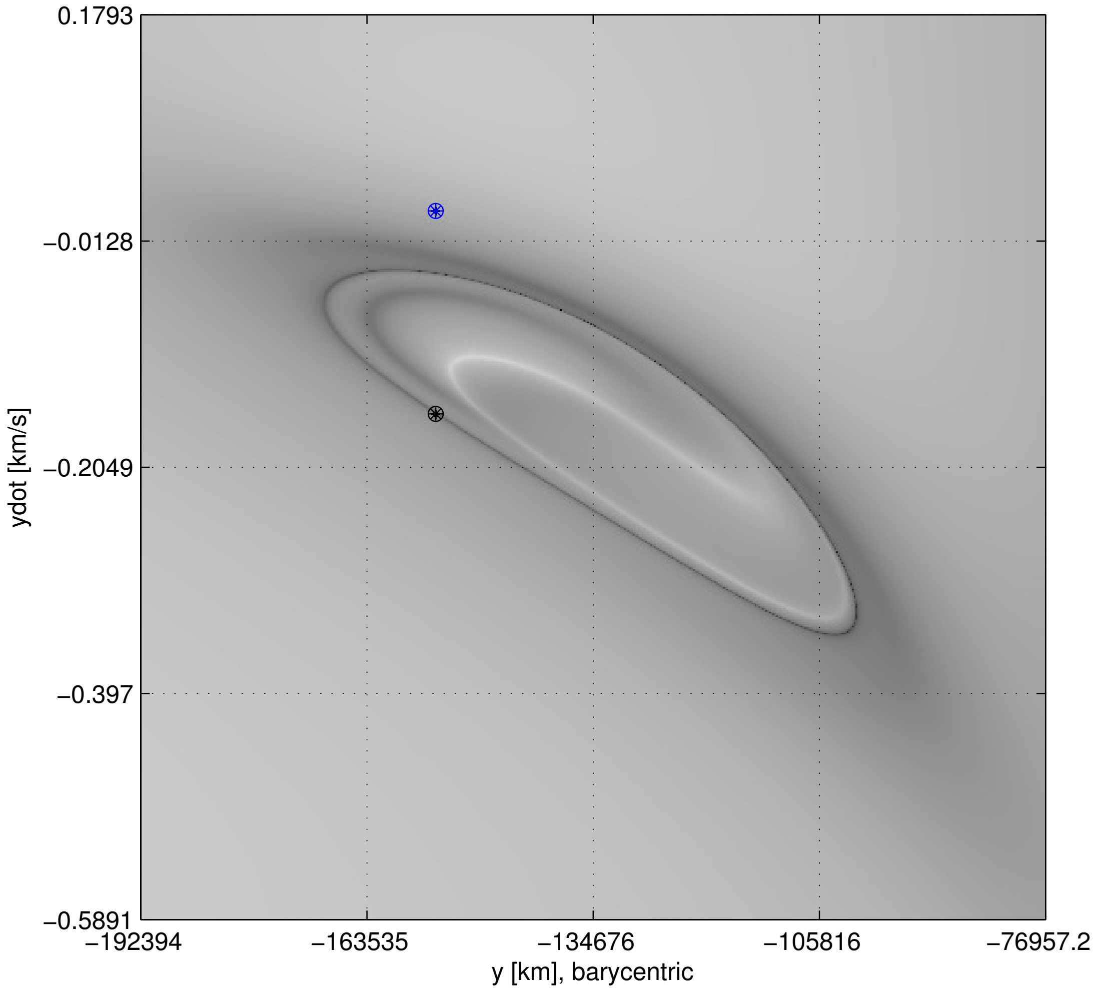


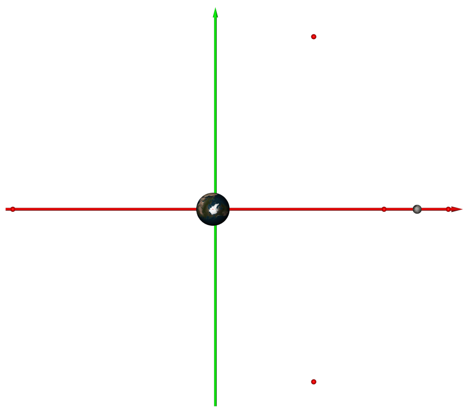
Hohmann – barycentered inertial
Hohmann – barycentered rotating

Section:
$-250{,}000 \text{ km} < y < -20{,}000 \text{ km}$
$-0.9733\frac{\text{km}}{\text{s}} < \dot{y} < 0.5635\frac{\text{km}}{\text{s}}$



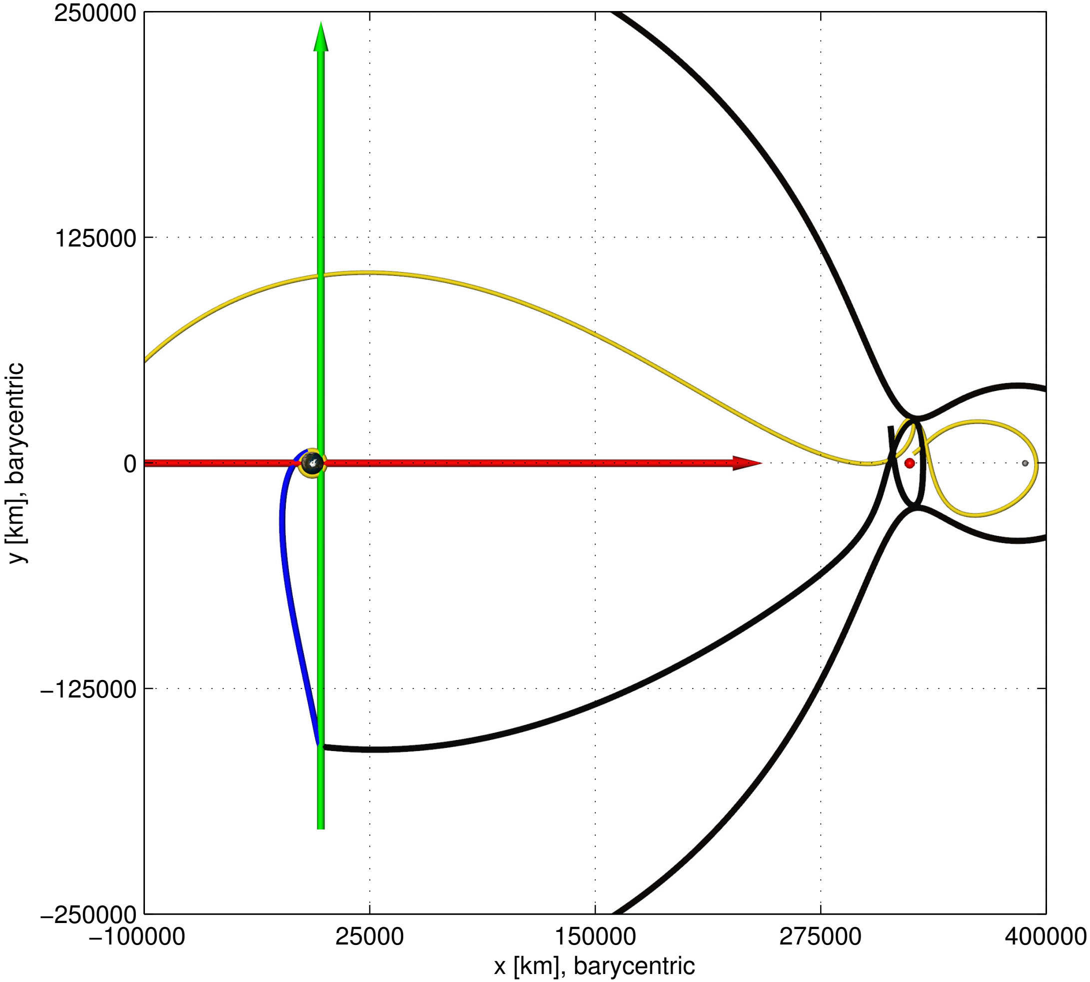
Hohmann
L_1 Lyapunov stable manifolds; $C = C_{L_2}$

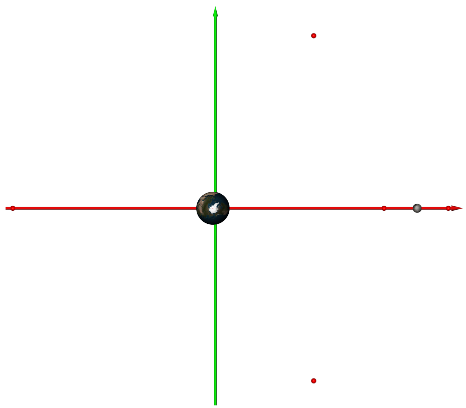




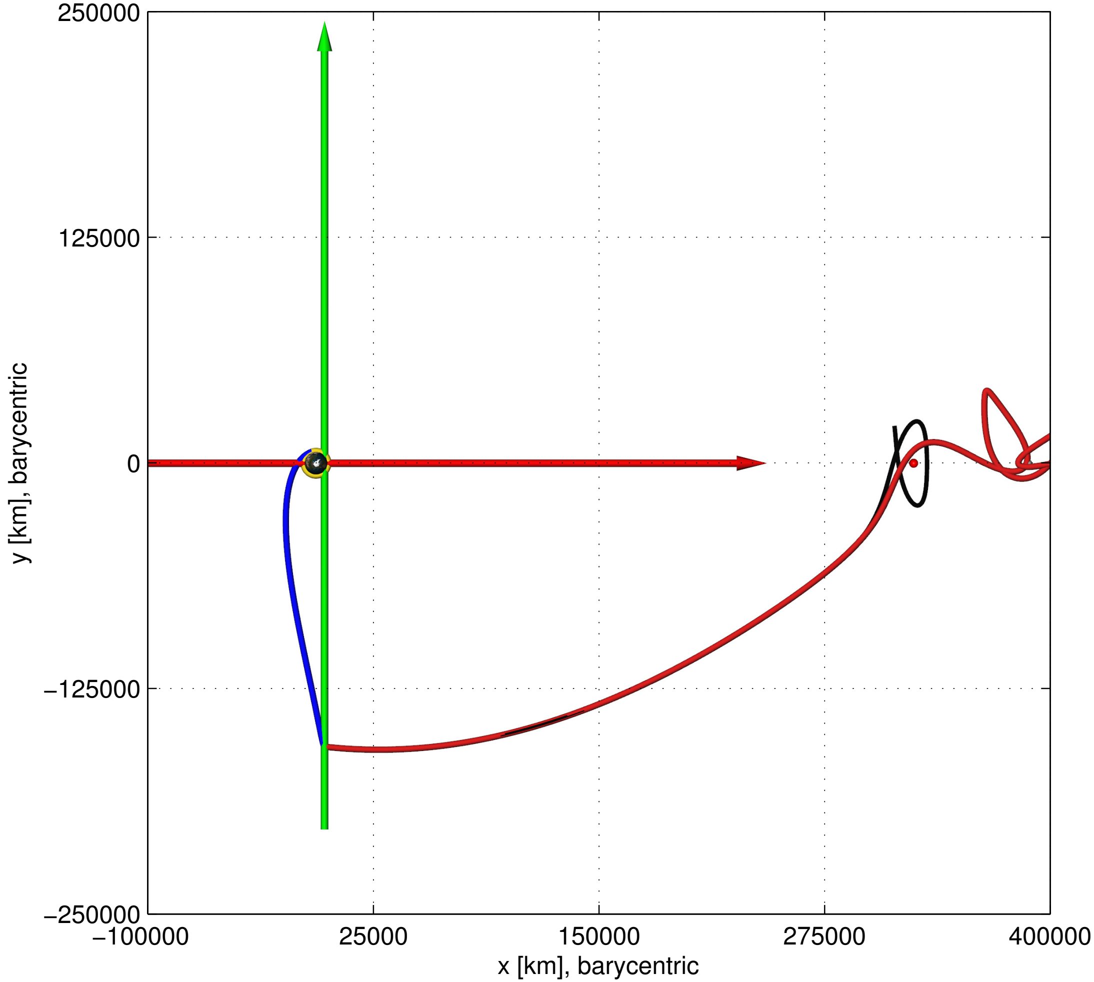
Hohmann state projection

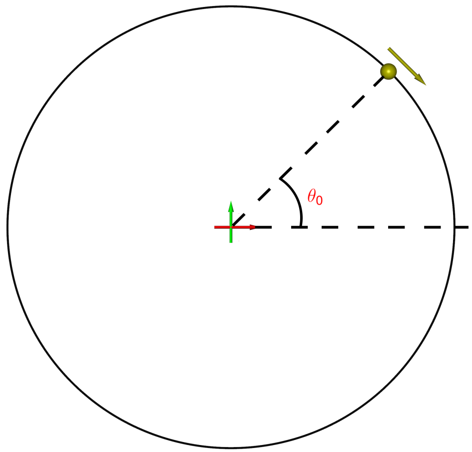
Map state near structure





$\dot{y} \approx -0.1546,$
$\dot{y} = -0.1550,$
$\dot{y} \approx -0.1561$

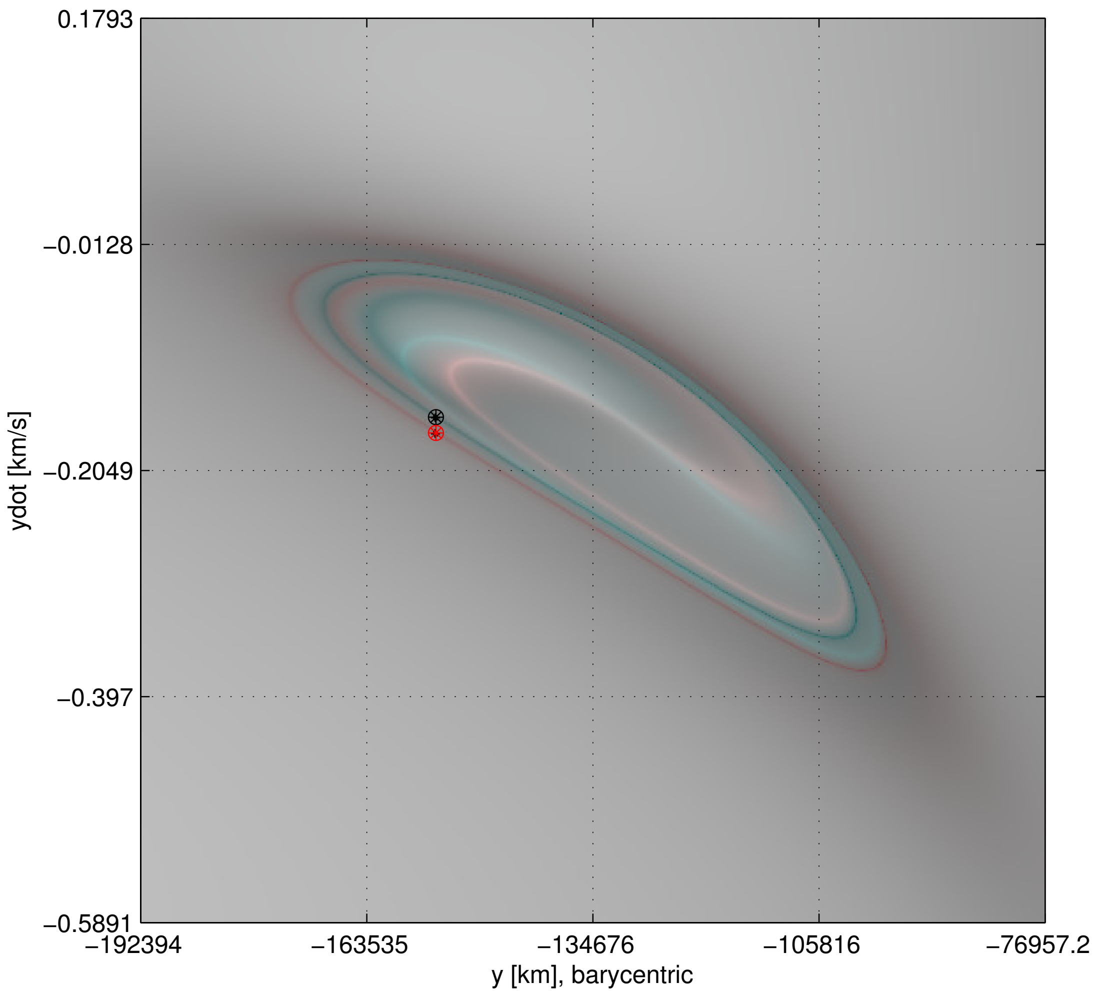


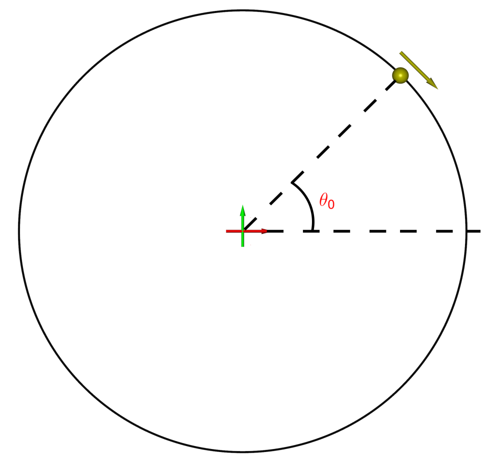


CRP: $\dot{y} = -0.1550$,

Same state in 4BP;

$\theta_0 = 0.25\pi$

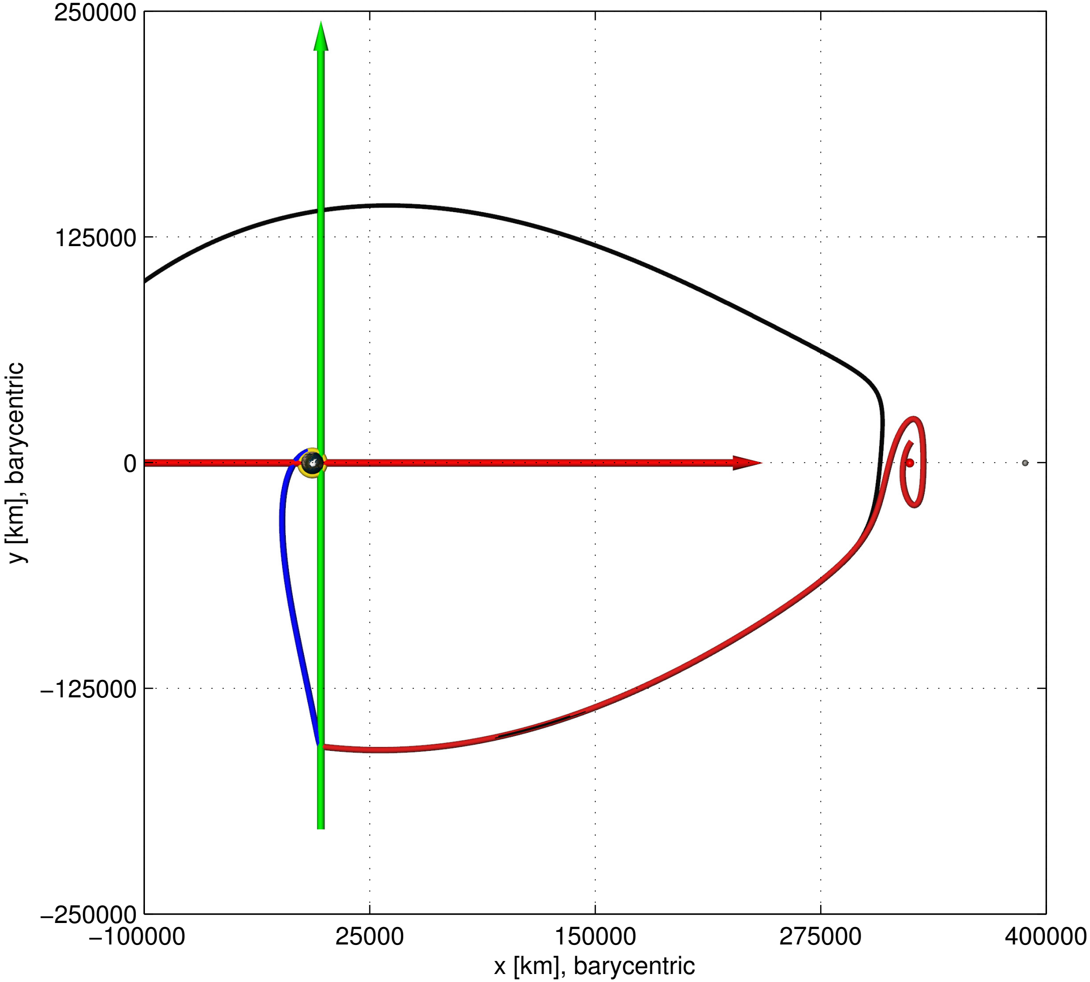


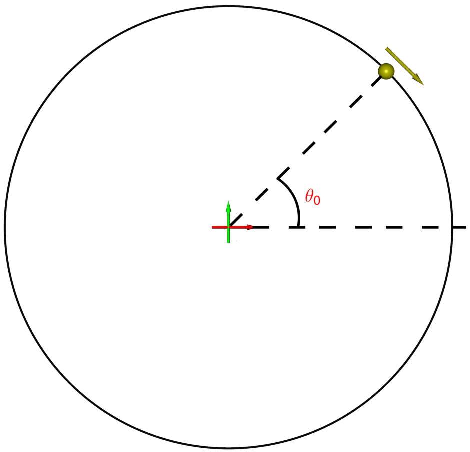


State in CRP,

New state for 4BP;

$\theta_0 = 0.25\pi$

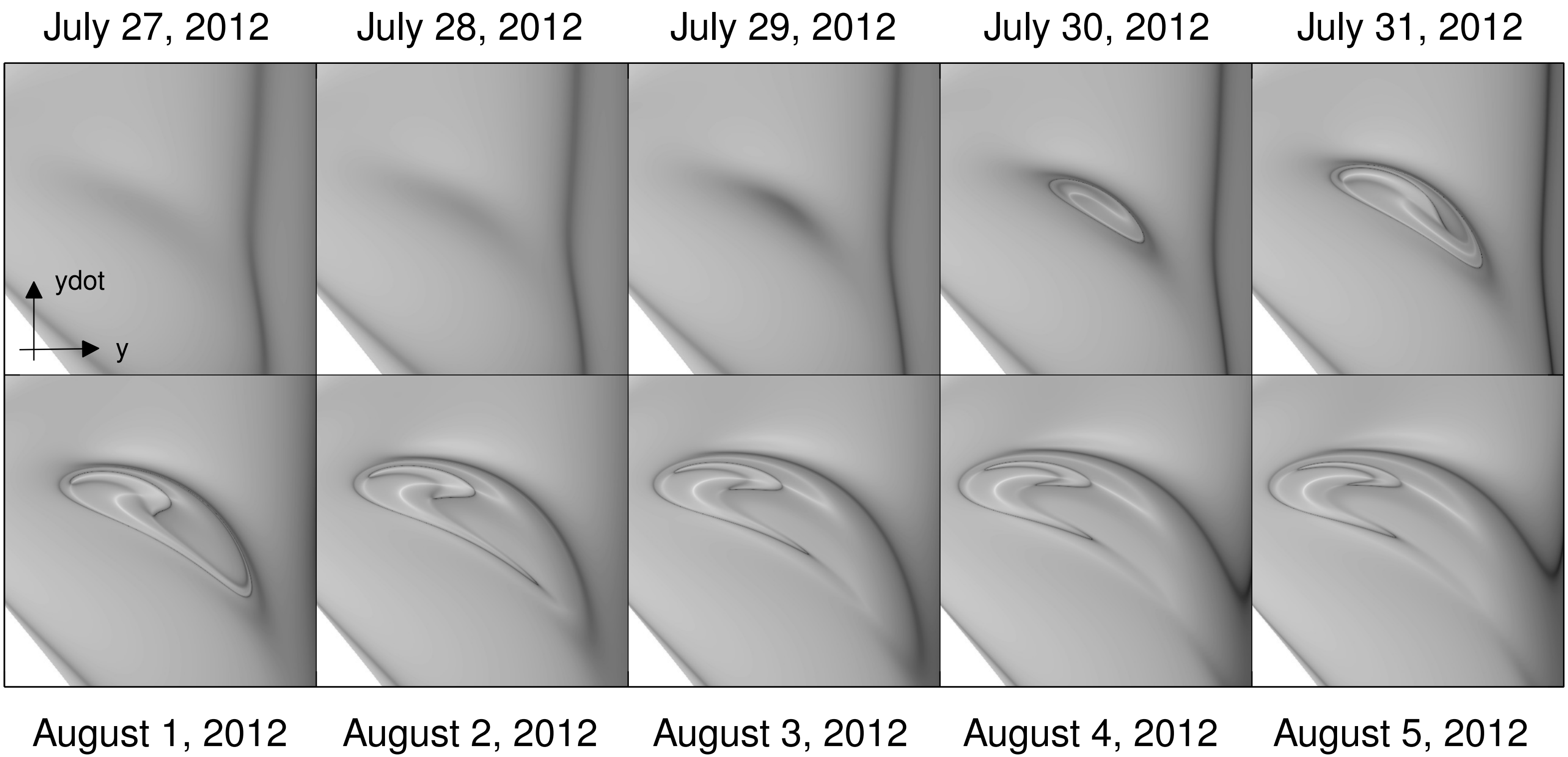


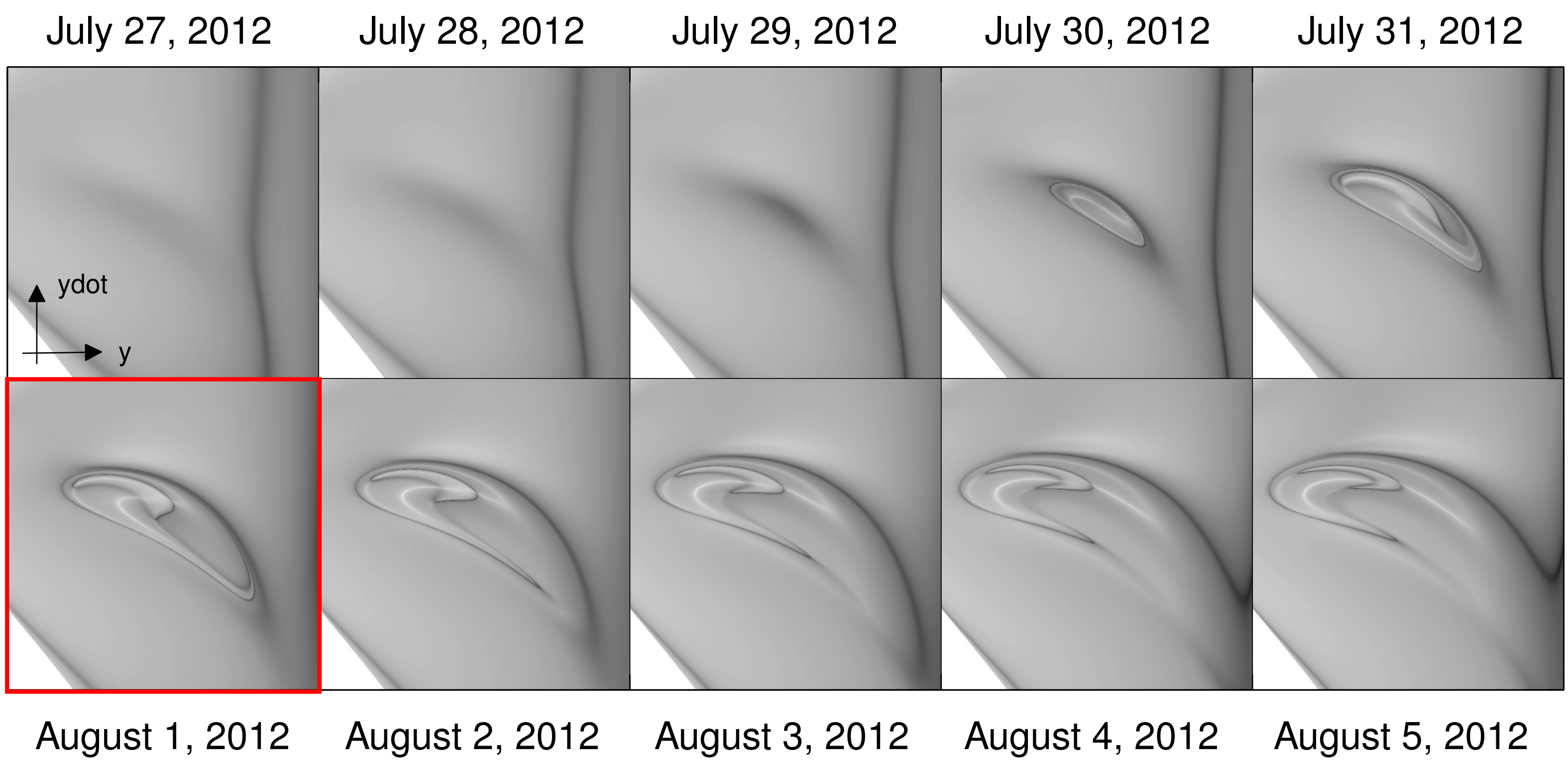


New state evolved in CRP,

New state in 4BP;

$\theta_0 = 0.25\pi$


July 27, 2012
July 28, 2012
July 29, 2012
July 30, 2012
July 31, 2012
ydot
y
August 1, 2012
August 2, 2012
August 3, 2012
August 4, 2012
August 5, 2012


July 27, 2012
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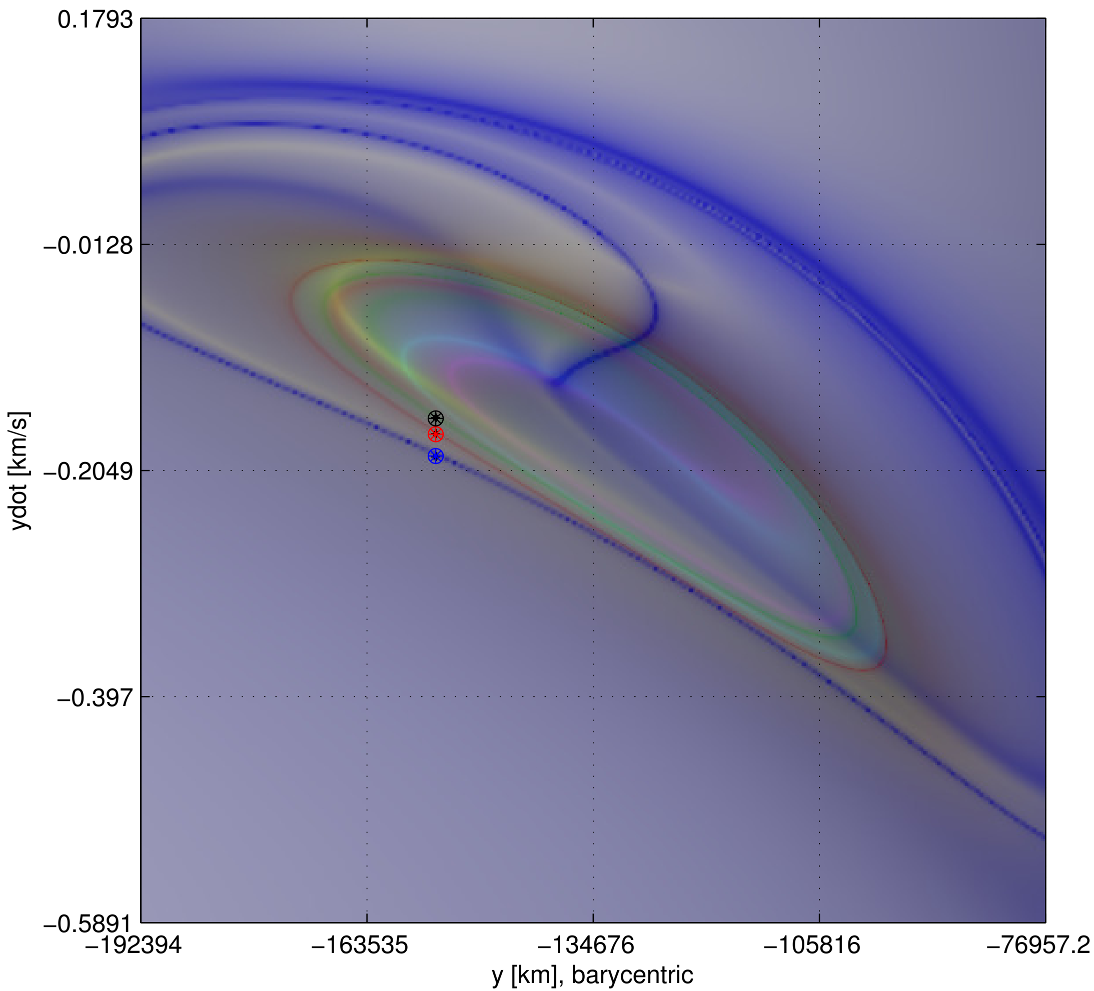
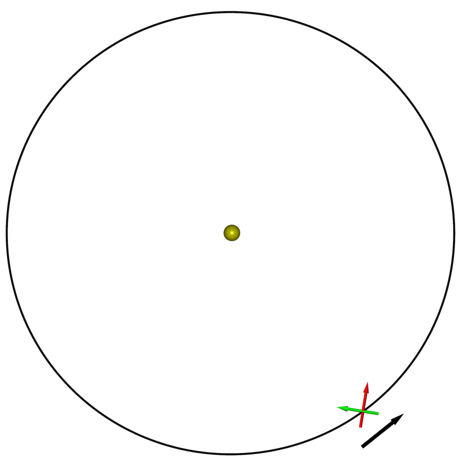
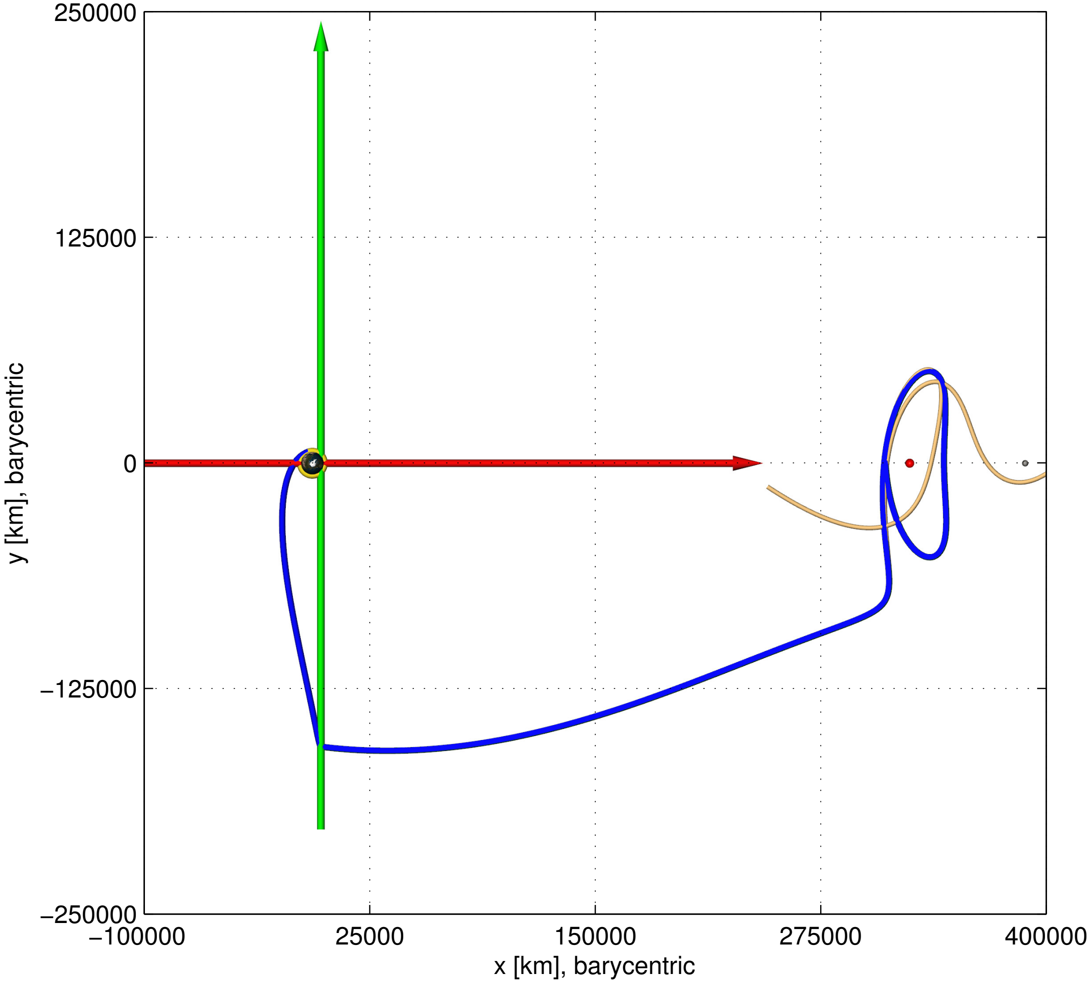
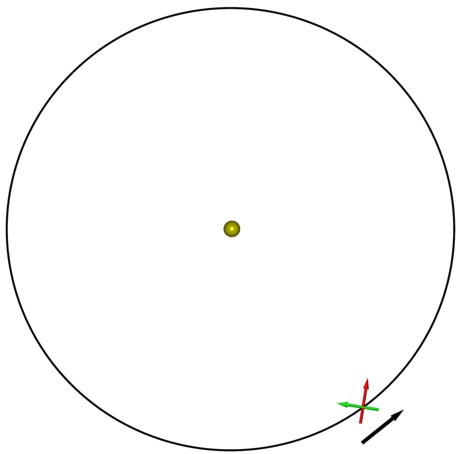
July 30, 2012
July 31, 2012
ydot
y
August 1, 2012
August 2, 2012
August 3, 2012
August 4, 2012
August 5, 2012





State in CRP,
State in 4BP,
State in SEM; Aug. 1





$\dot{y} \approx -0.1883$,
$\dot{y} = -0.1881$,
$\dot{y} \approx -0.1868$

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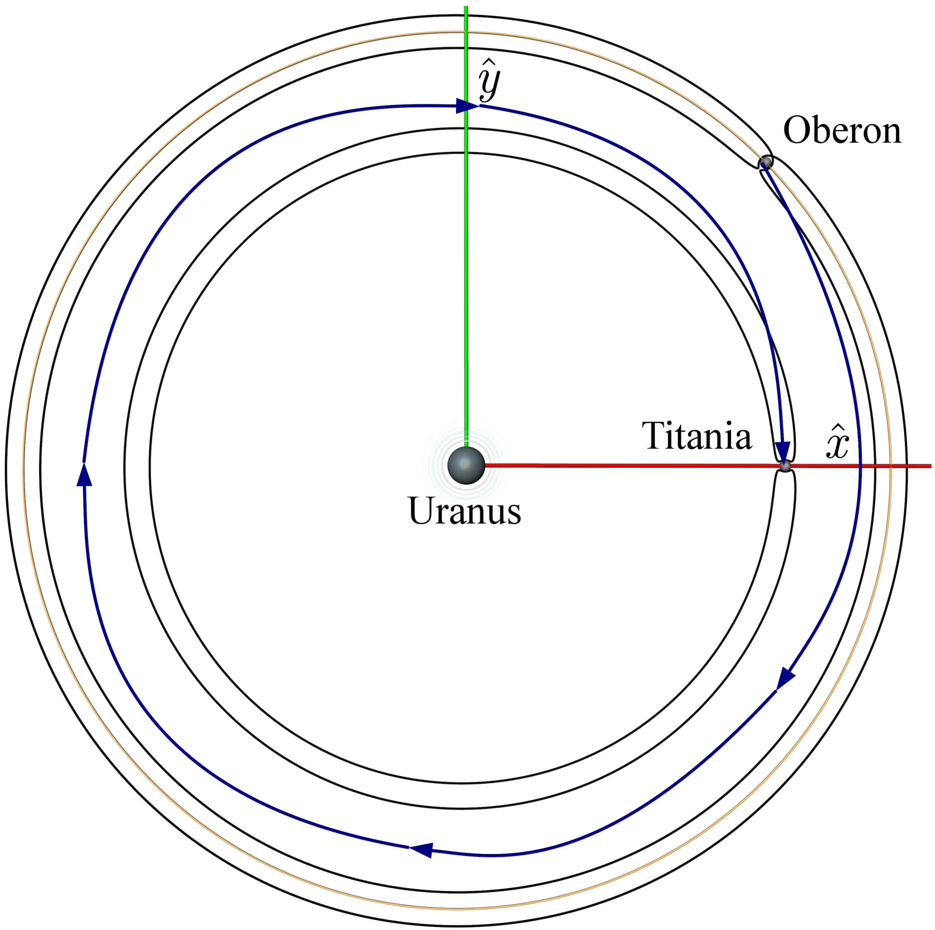
- Objective: Revolution about Oberon, transfer from Oberon to Titania, revolution about Titania
- Find transfer using forward/backward advection of flow control segments



- Identify states near each moon to evolve and find intersections
  - Define sections guided by CRP Zero Velocity Curves (ZVC)
  - Construct FTLE maps on sections
  - Identify and refine high-valued FTLE points
  - Augment maps with LCS candidate strainlines
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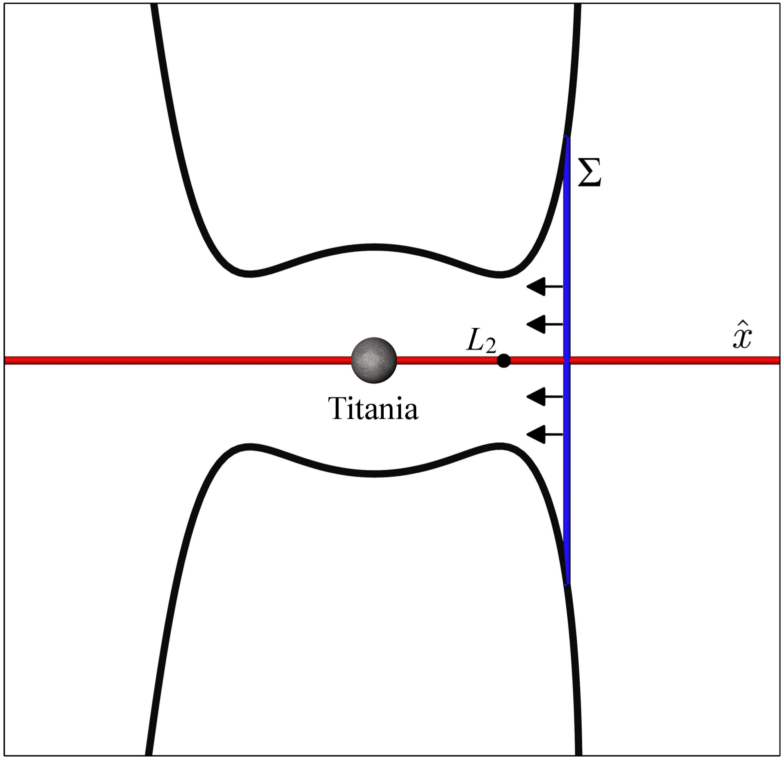
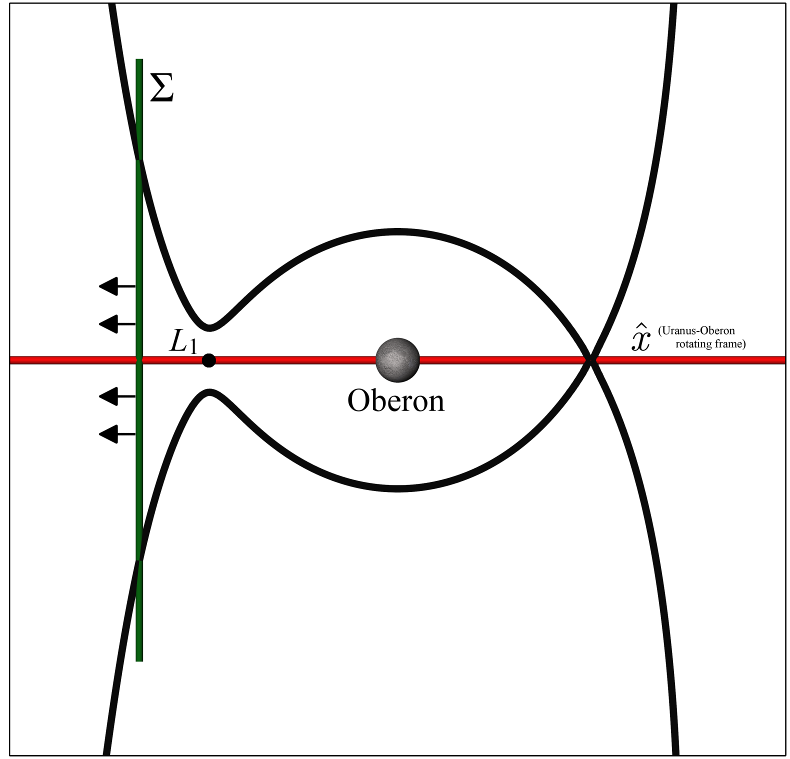
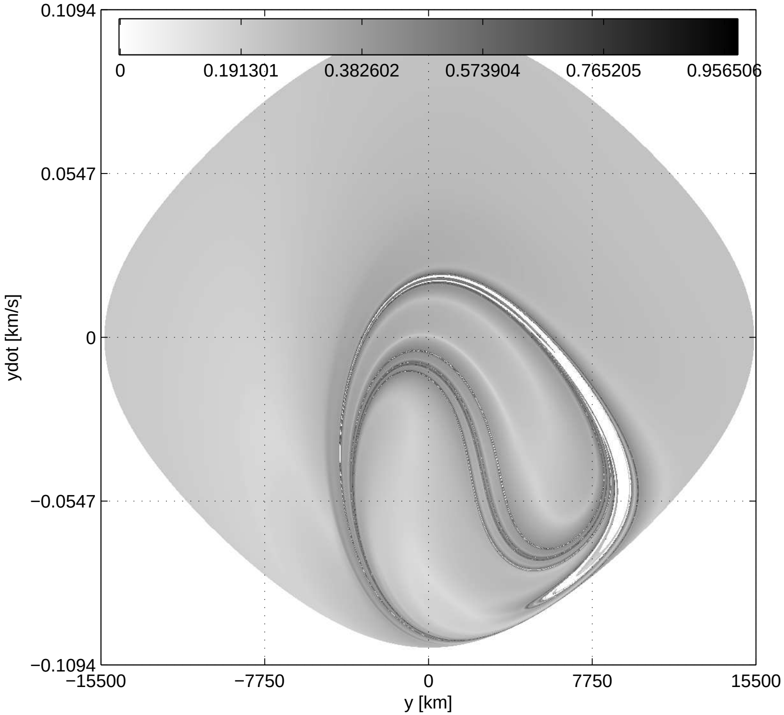




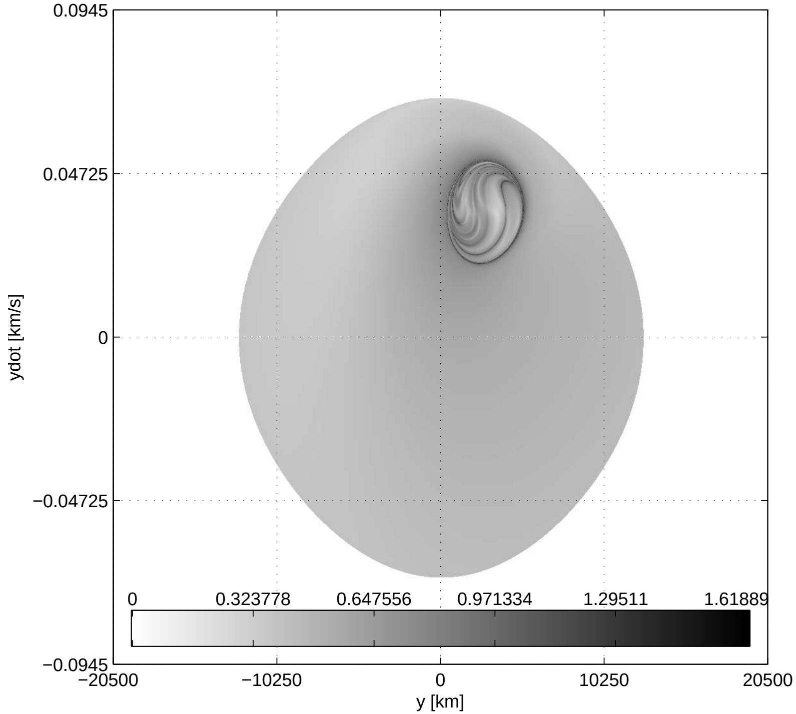
- $\Sigma : x = 1.03$ nd,
  $-0.035 \leqslant y \leqslant 0.035$ nd,
  $-0.03 \leqslant \dot{y} \leqslant 0.03$ nd
- $\dot{x}$ from C selected to point toward Titania in forward time

- $\Sigma : x = 0.97$ nd,
  $-0.035 \leqslant y \leqslant 0.035$ nd,
  $-0.03 \leqslant \dot{y} \leqslant 0.03$ nd
- $\dot{x}$ from C selected to point away from Oberon in forward time

- Main internal structures: trajectories that flow toward respective moon
- ‘Swirls’: subsequent behavior (remain near moon or depart)
- Solid white regions: values set to zero (outside CRP ZVC)

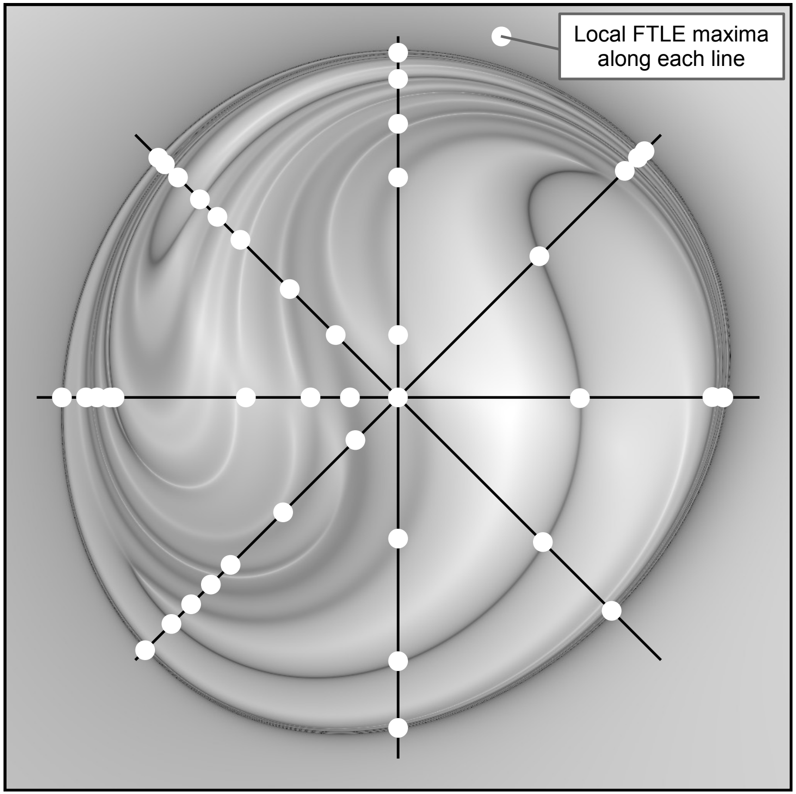


Forward time → Titania in 4BP
10 nd time steps (∼13.8 days)

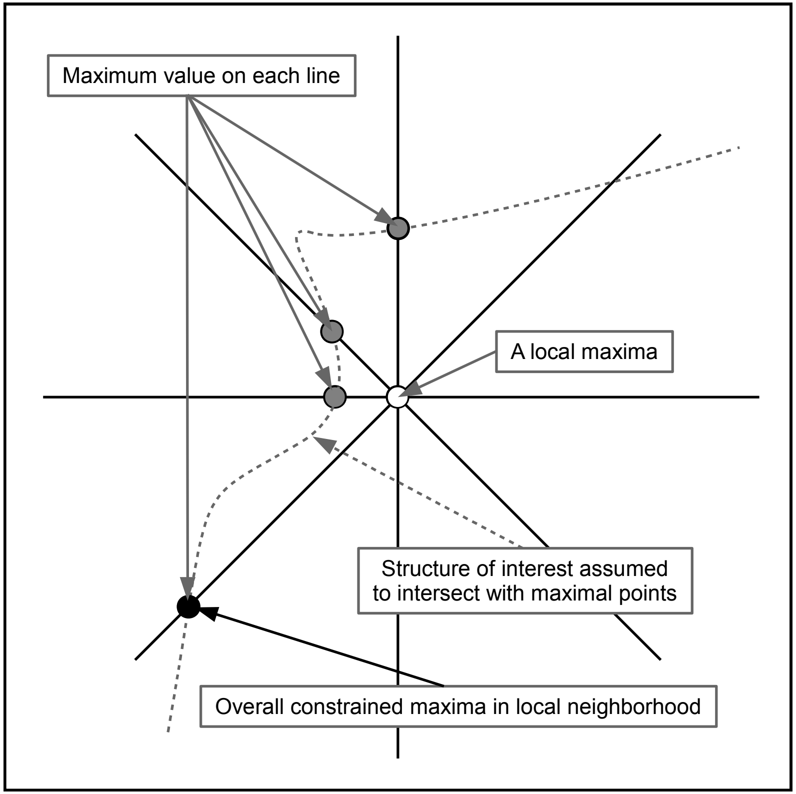


Backward time → Oberon in 4BP
10 nd time steps (∼21.4 days)

- Potential LCS member points may reflect high FTLE values
- Identify points to seed LCS strainlines

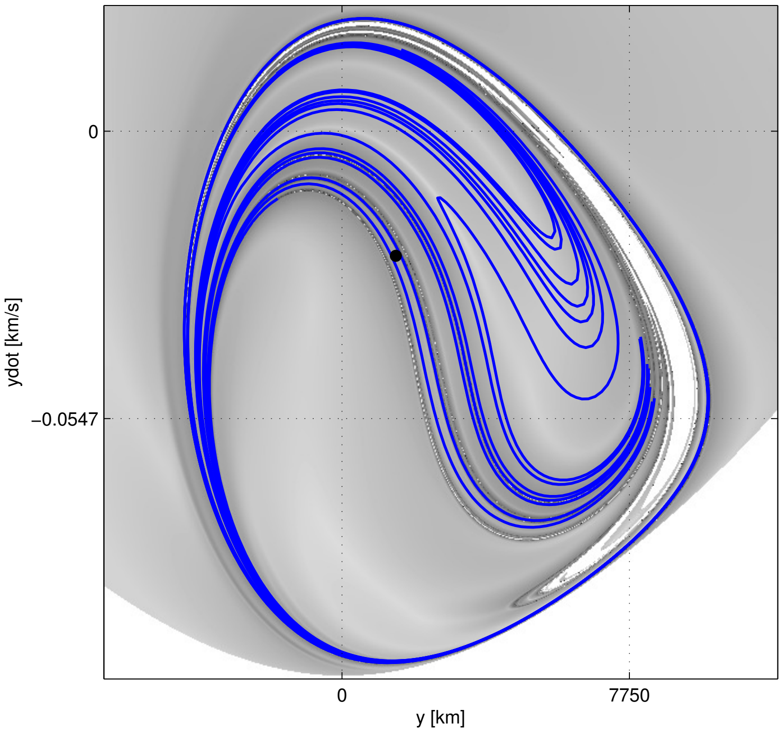


Automated radial search for maxima

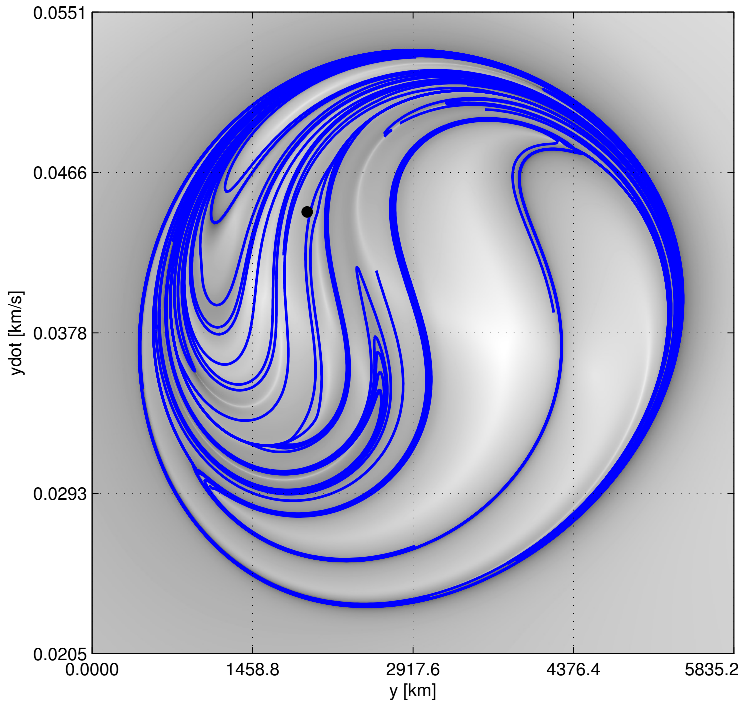


Point refinement in local neighborhood

- Seeded from refined FTLE points
- Use additional vector information from CGST to propagate curves
- Select states to construct trajectory end segments

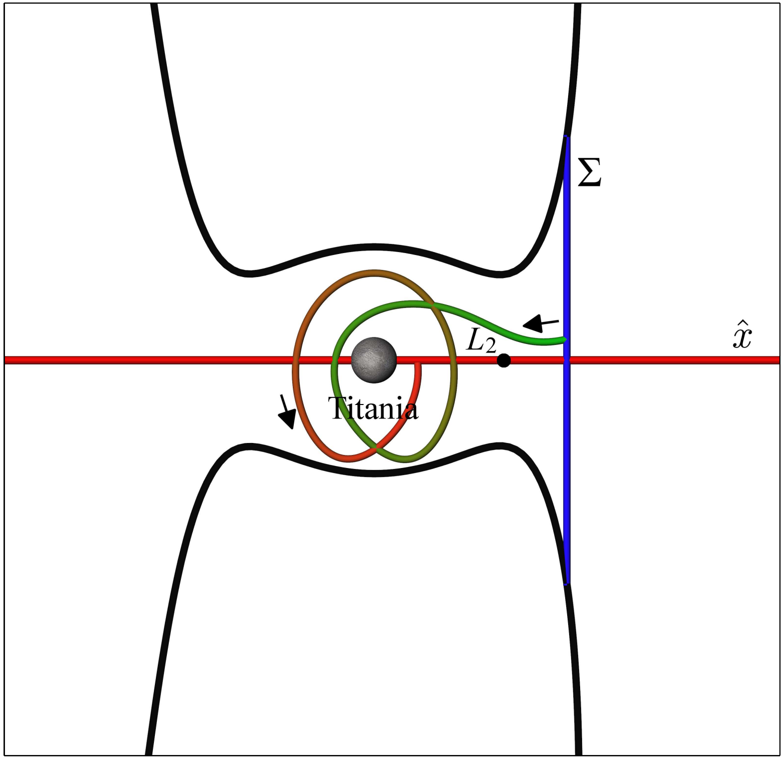


Strainlines on Titania map

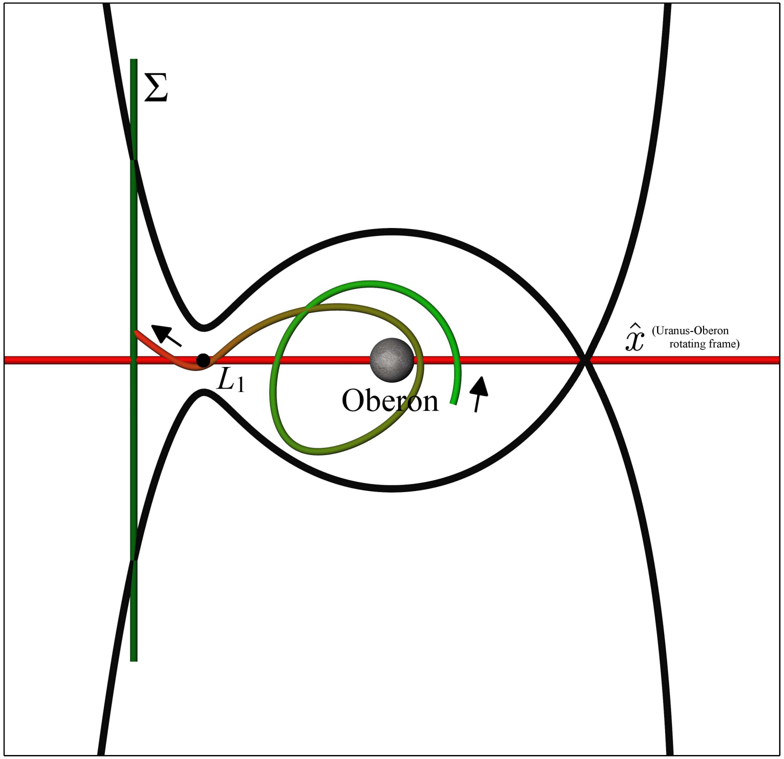


Strainlines on Oberon map

Transfer "Bookends"



- Terminal phase of the design; revolution about Titania
- Forward time evolution of state from Titania FTLE map



- Origin of the design; revolution about Oberon
- Backward time evolution of state from Oberon FTLE map

Process

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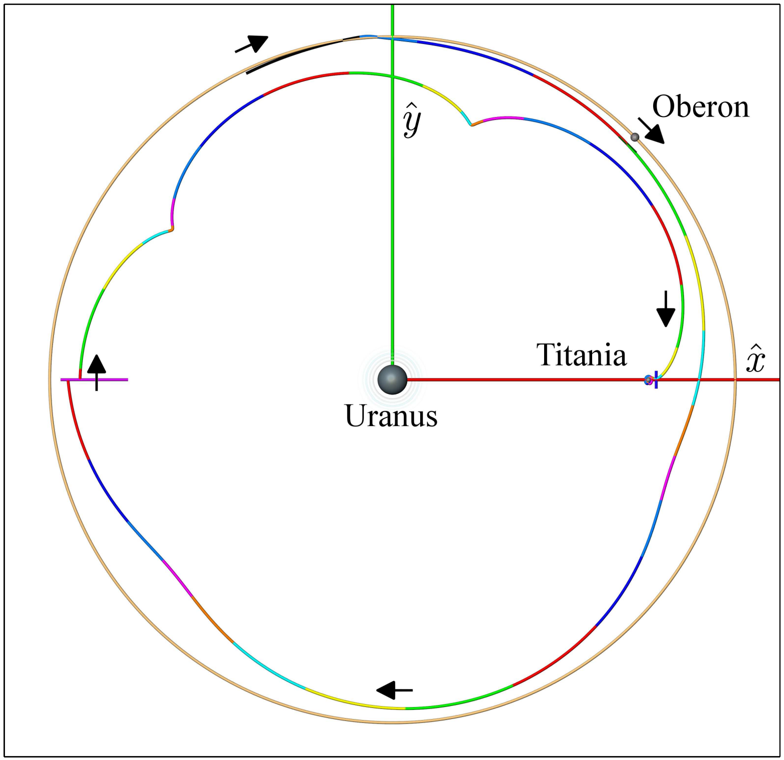
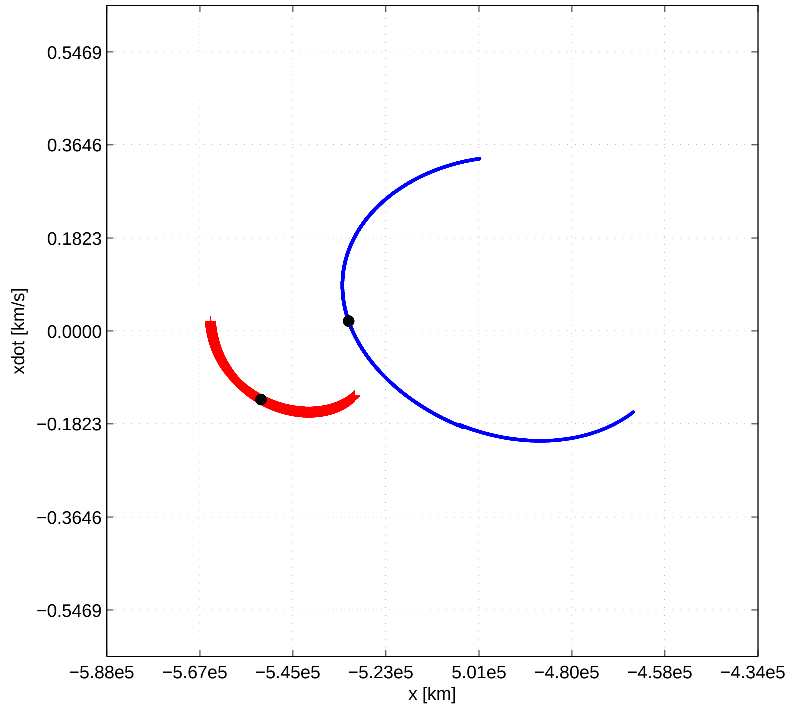
- Section defined on negative *x* axis (magenta)
- Trajectories use rotating colors, change every time step (∼1.38 days)
- Crossings from Oberon in forward time, from Titania in backward time



First forward and backward crossings



FCS images on the map (Oberon, Titania)
• central trajectory

Process

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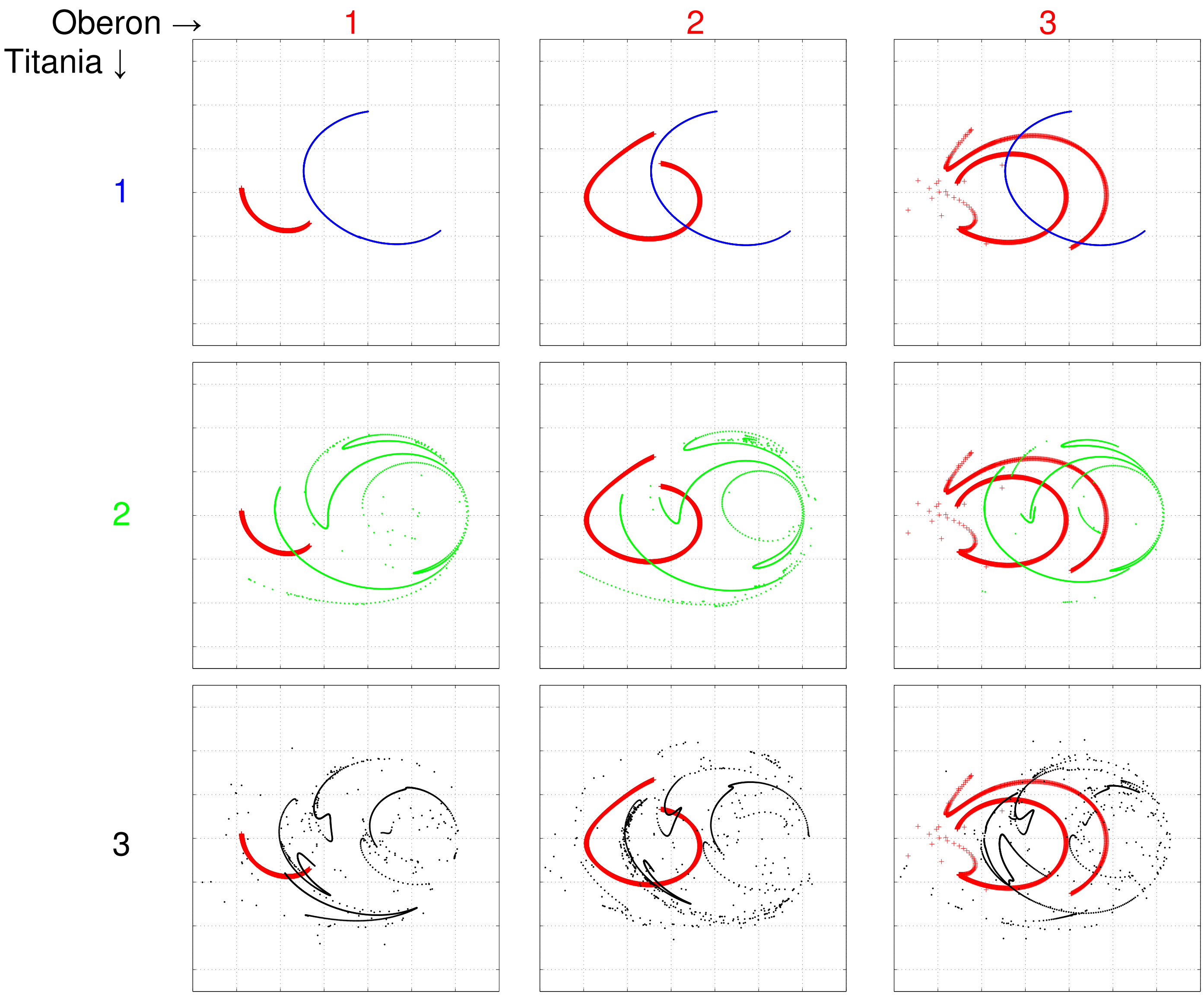
FCS Section Crossings for Various Revolutions



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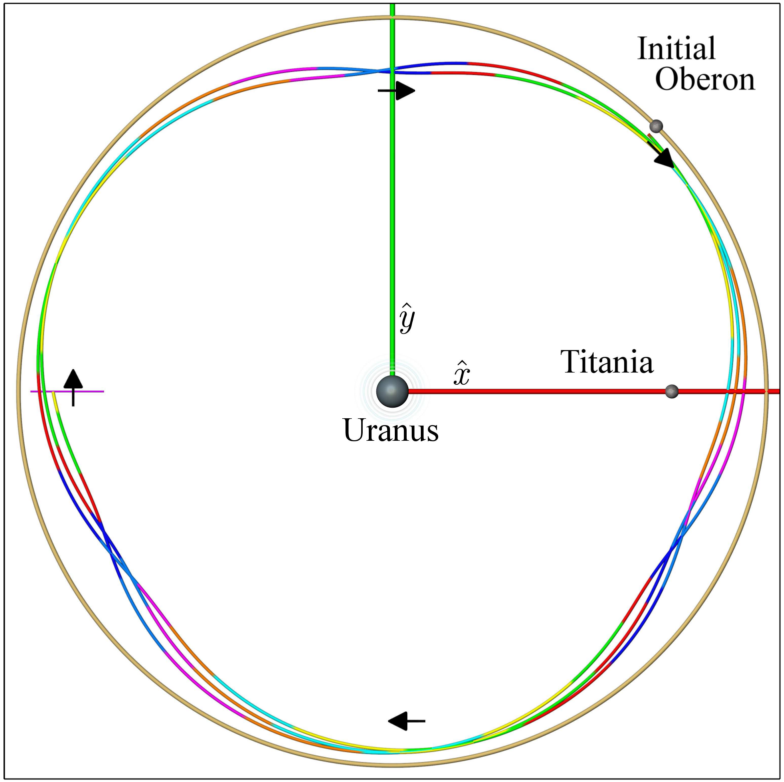
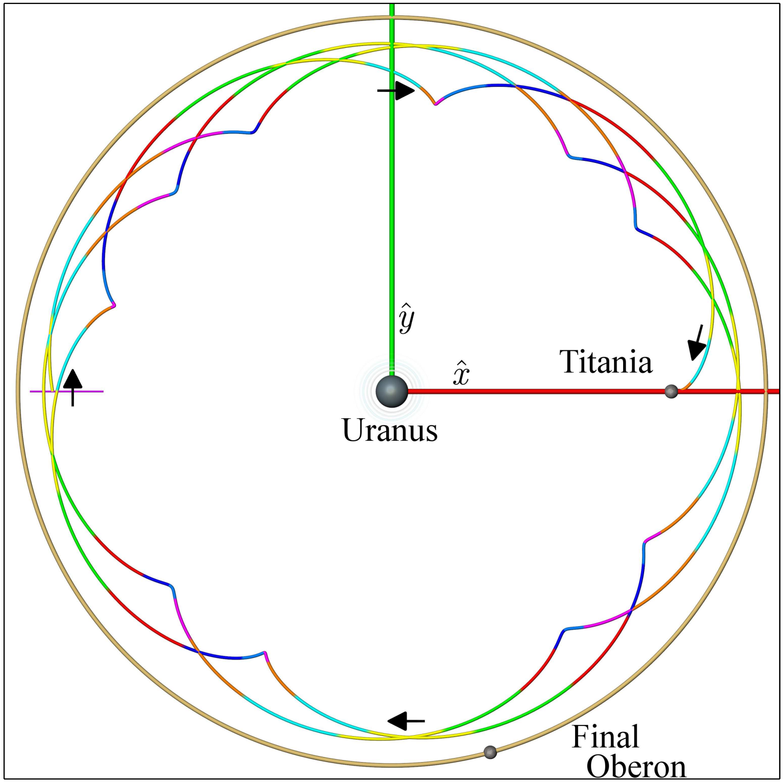
- Oberon segment (forward time) always starts with Oberon at $\frac{\pi}{4}$ radians
- Titania segment (backward time) initializes with estimated geometry
- Use transfer time-of-flight to correct Oberon phasing at Titania arrival
- Iterate to find intersection that satisfies geometry constraints

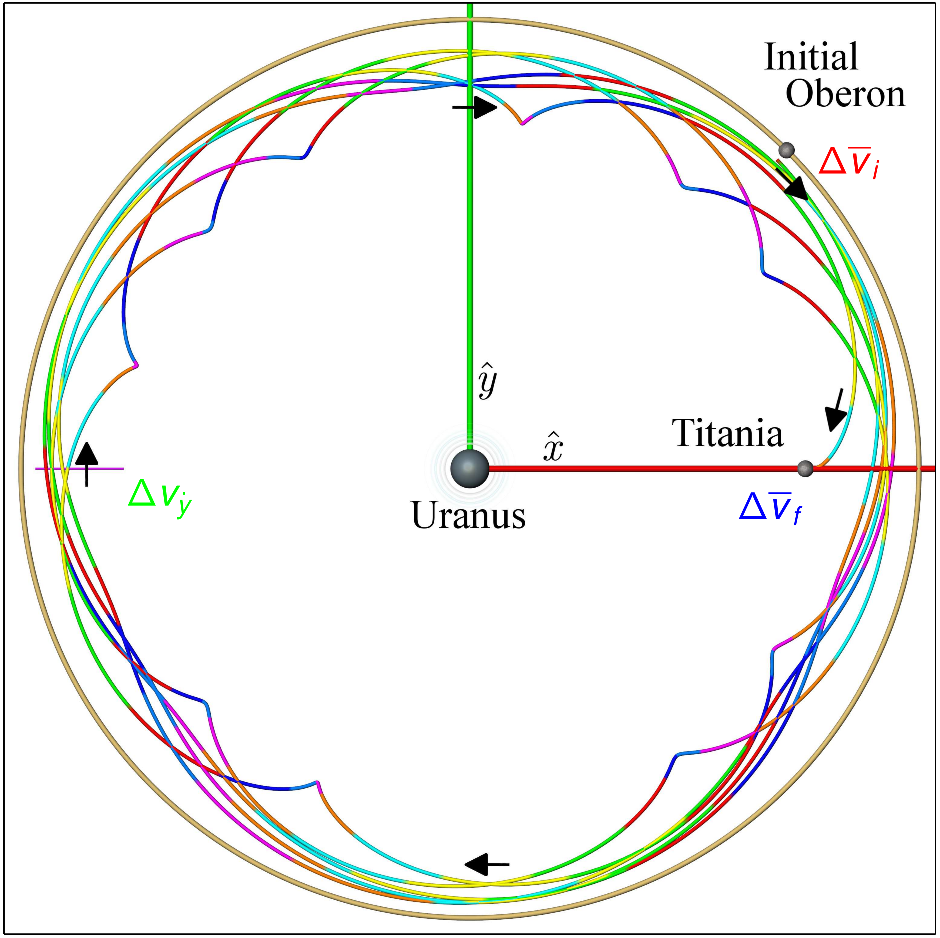


Three forward crossings



Three backward crossings

- Time-of-flight: 204.55 days
- Maneuver cost: $\Delta\bar{v}_i + \Delta v_{\dot{y}} + \Delta\bar{v}_f = 125.45$ m/s
- Complex solution using fairly straightforward process



Maneuver Costs and Times of Flight

	[m/s], [days]	[m/s], [days]	[m/s], [days]
BWD ↓ FWD →	1	2	3
1	No Intersections	182.97, 81.56	173.93, 116.41 179.46, 125.81 123.43, 119.31 163.93, 111.23
2	168.39, 91.23	142.71, 126.09 219.77, 138.94	133.74, 160.73 181.23, 186.53 167.90, 181.32 164.77, 180.81 195.42, 172.95
3	161.20, 134.90 153.86, 143.18 180.54, 142.90	126.82, 178.13 144.94, 177.99 206.28, 190.69 209.84, 190.51 208.14, 190.14	125.45, 204.55 170.13, 212.14 160.14, 247.18 182.08, 241.87 130.89, 232.53 179.08, 224.29

- Lowest propellant cost (Forward 3, Backward 1): **123.43 m/s**
- Shortest TOF (Forward 2, Backward 1): **81.56 days**
- **Hohmann** transfer between departure and arrival radii: **391.53 m/s**, **5.46 days**
- **Lambert-arc** with phasing (260.17°) of 'lowest propellant cost' case: **458.56 m/s**, **6.29 days**
- **Three-body** minimum change in velocity (from Jacobi constants): **150.99 m/s**
- The flow-based solutions have long time durations but cost less, in terms of fuel, than simple comparisons in the two- and three-body models. They also represent potential initial guesses for further improvement via numerical corrections in more realistic models.

$\Rightarrow$ Goal: Apply and extend flow-based tools for spacecraft trajectory design in multi-body regimes

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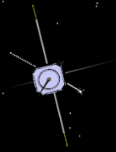
Flow-based strategies extend existing methods and supply new methods that enable:

⇒ Goal: Apply and extend flow-based tools for spacecraft trajectory design in multi-body regimes

Flow-based strategies extend existing methods and supply new methods that enable:

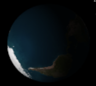
- Trajectory design and analysis in complex models incorporating many effects to reveal new insight
- Tools that apply across levels of model fidelity and highlight the contributions of particular components
- Decreases in the gap between preliminary design and workable solutions
- Identification of many, potentially better, options



Flow-Informed Strategies for Trajectory Design and Analysis

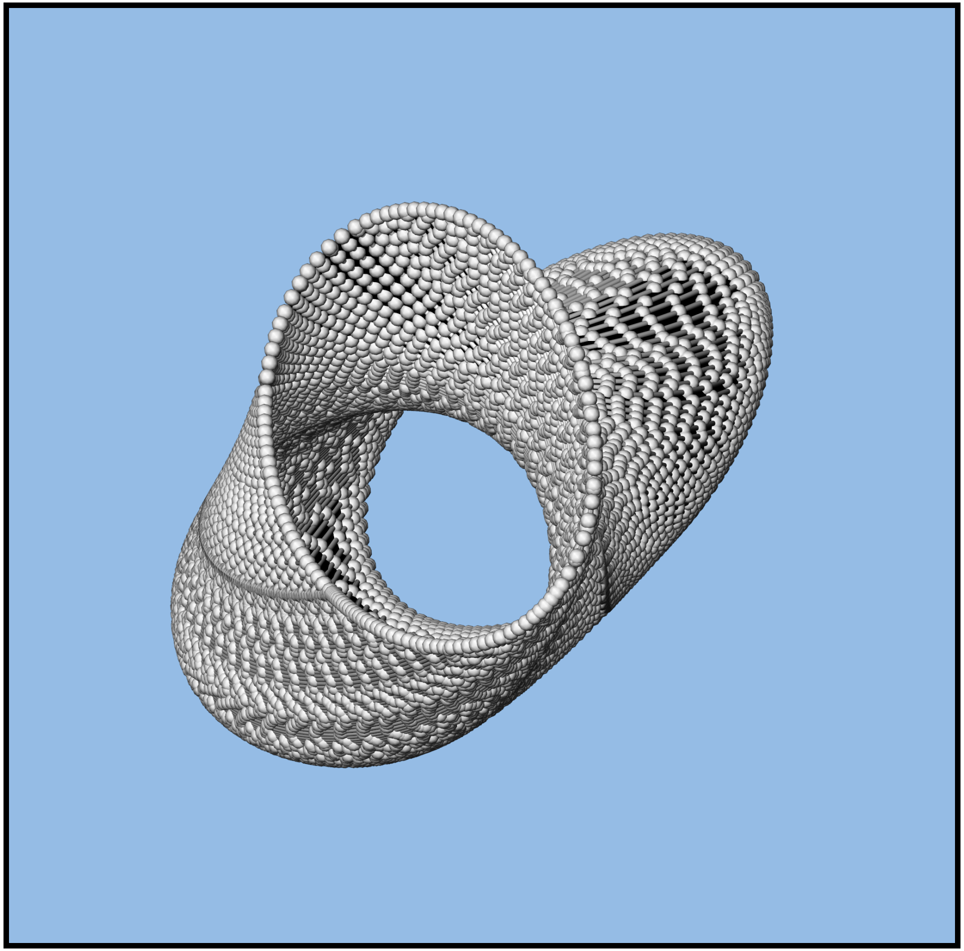
Computational Science & Engineering Seminar

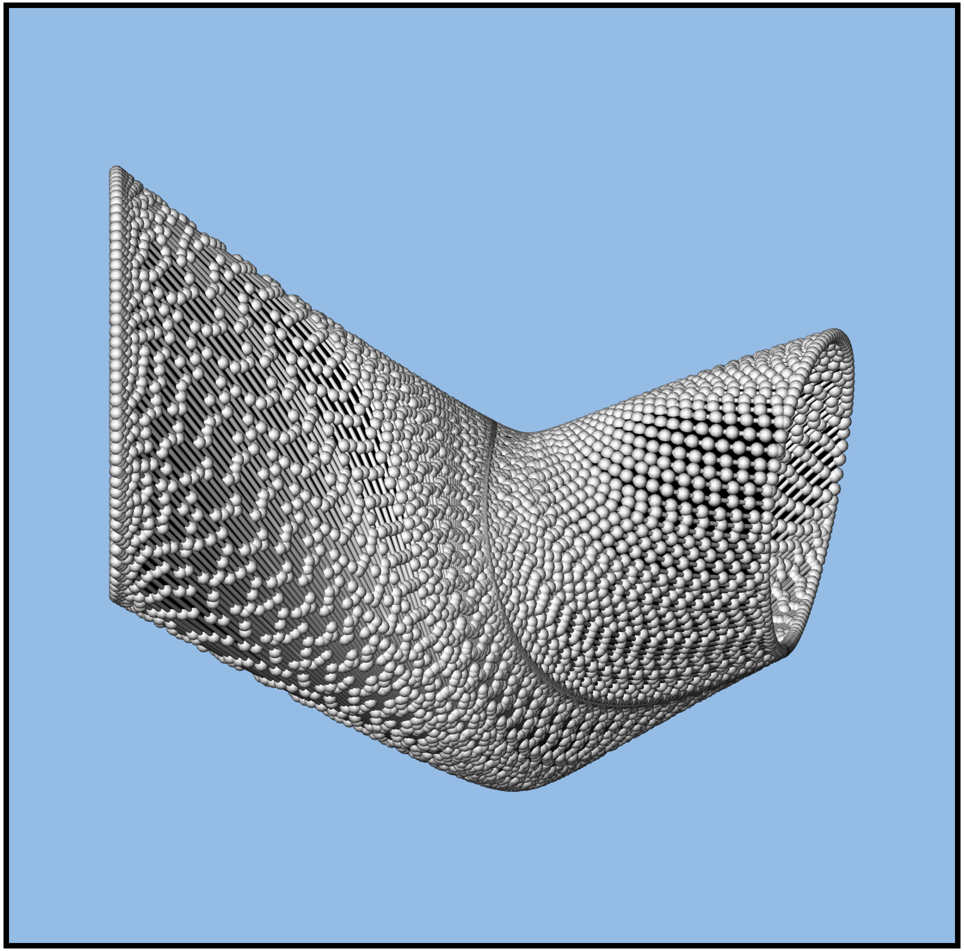
Cody R. Short



School of Aeronautics & Astronautics
Purdue University

February 4, 2015





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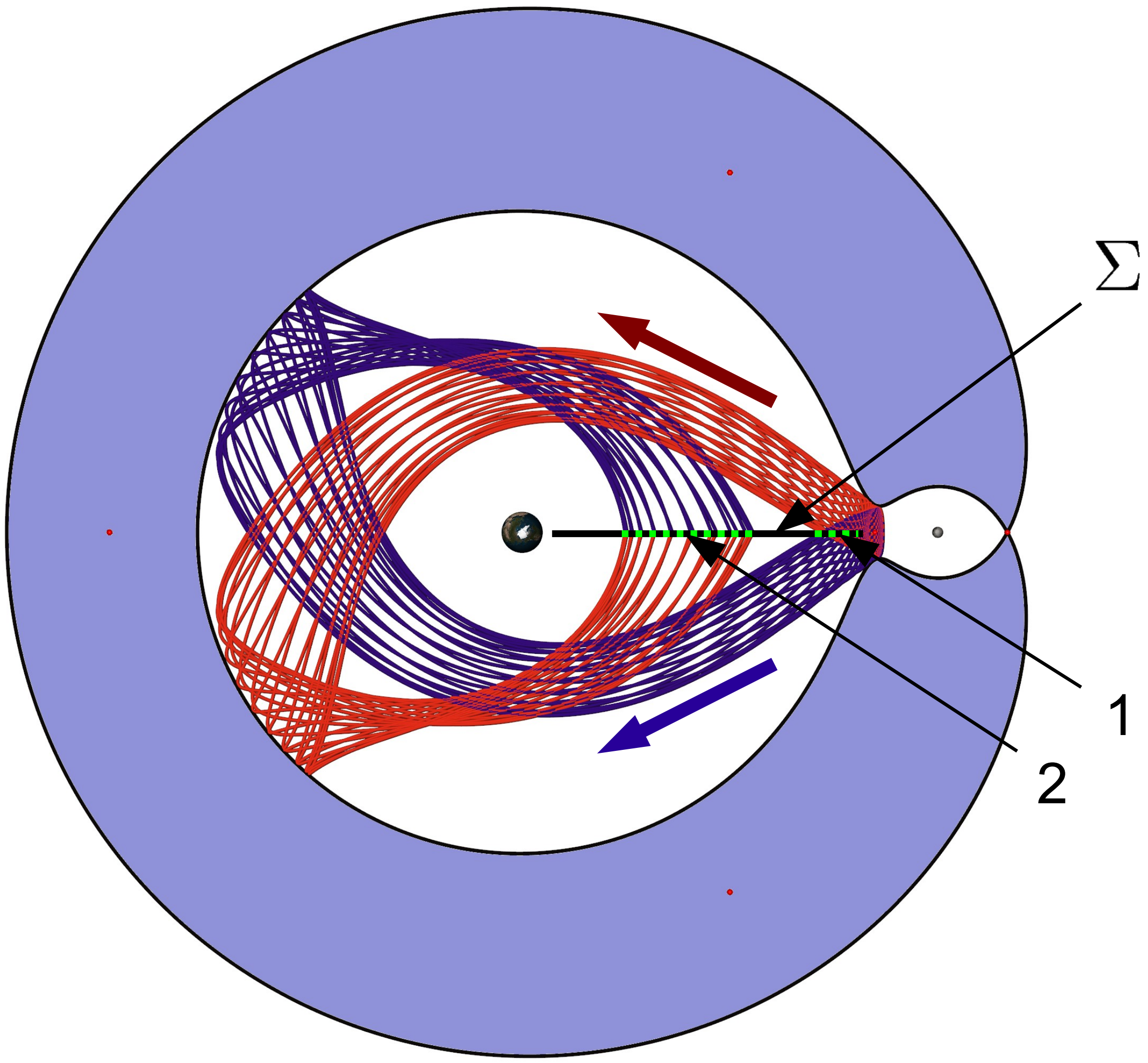
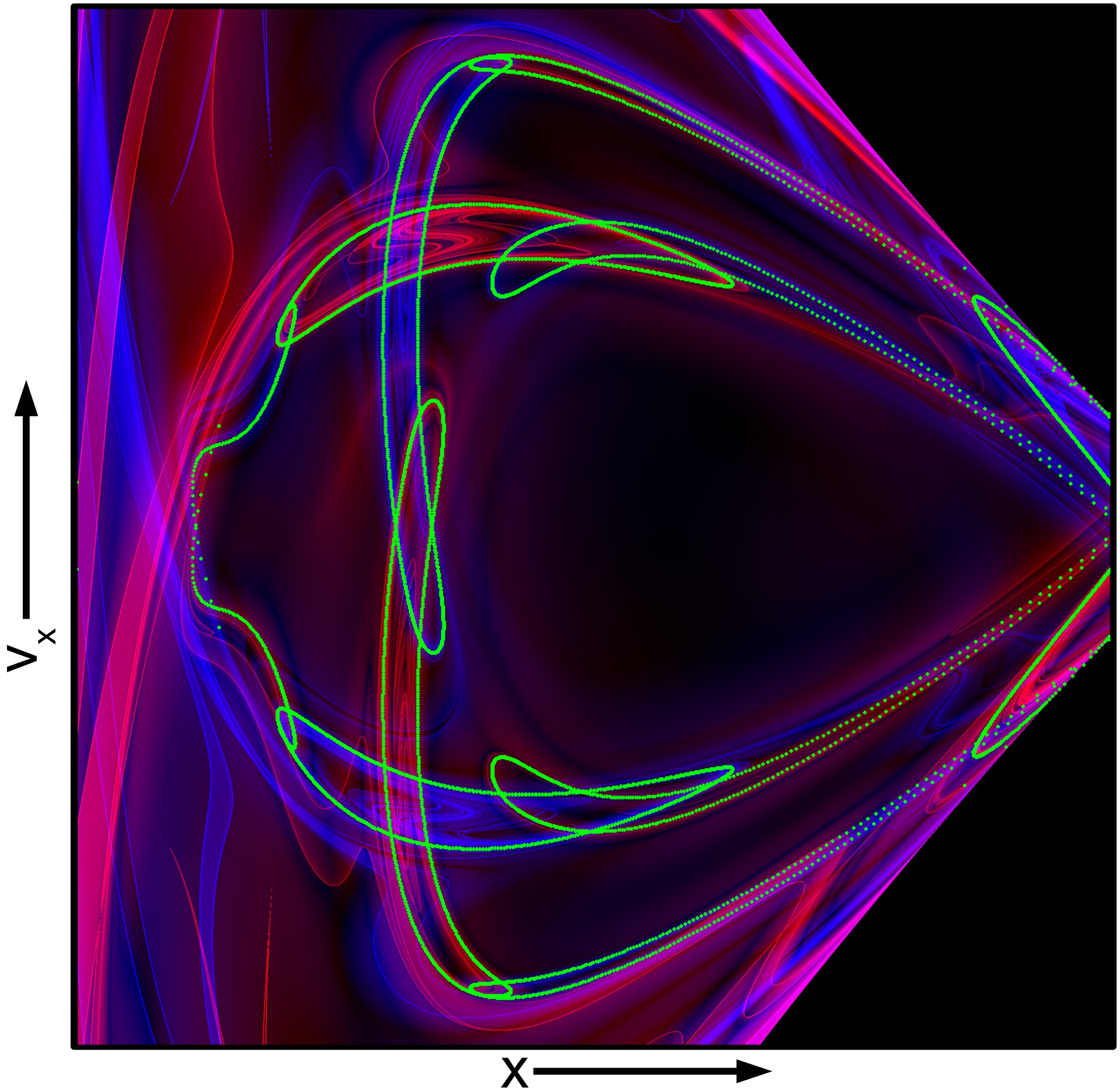
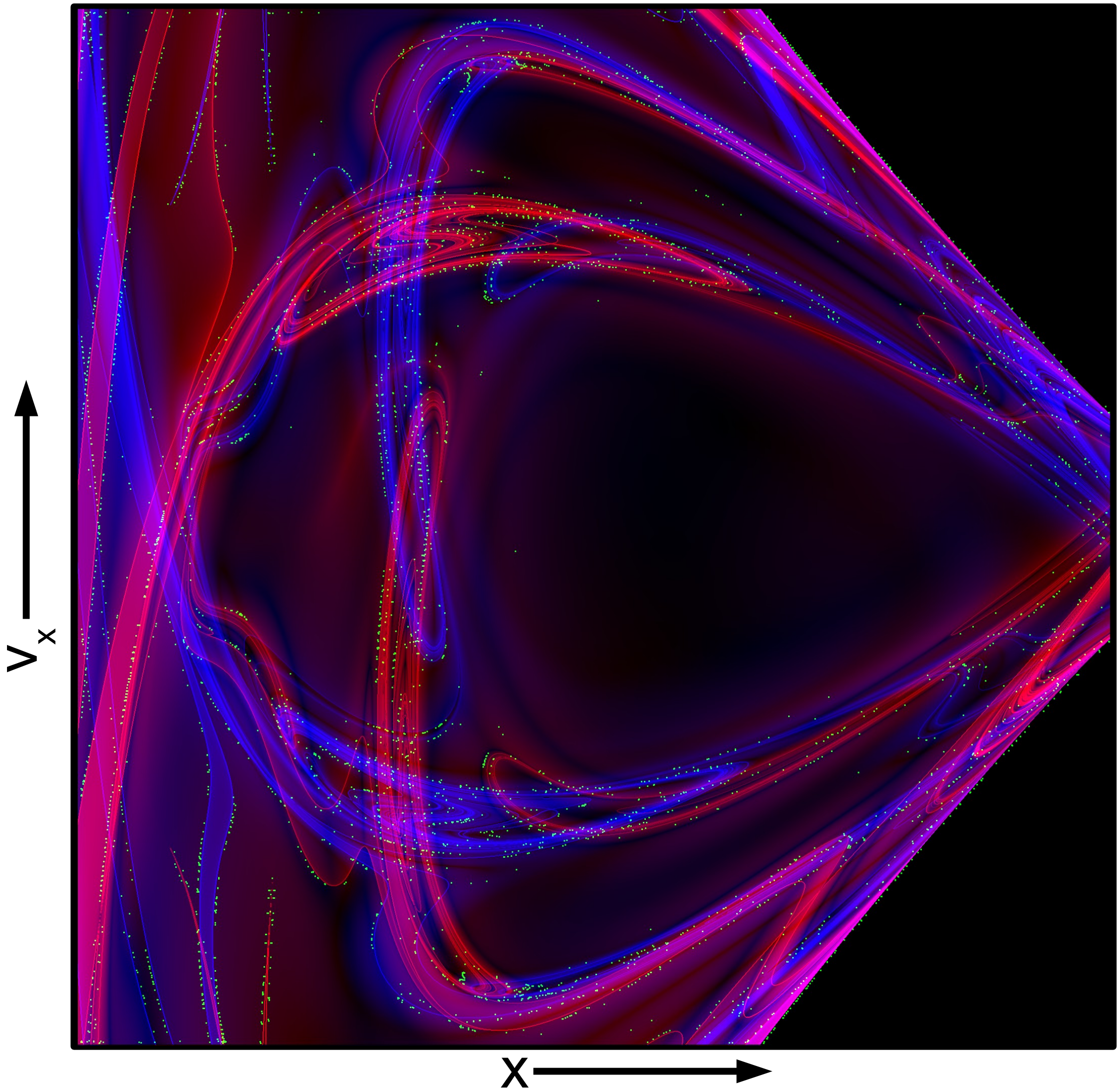
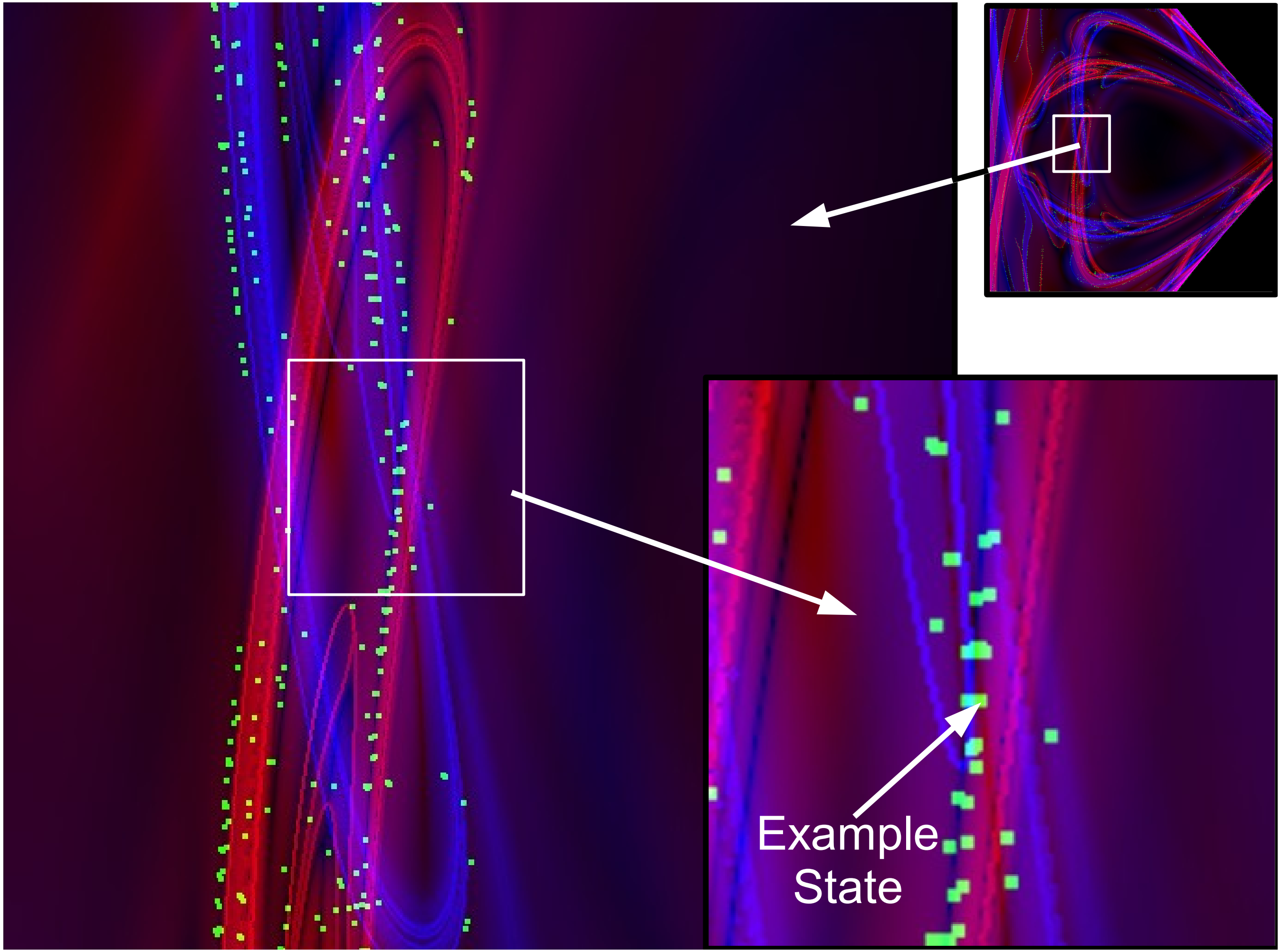

Σ
1
2

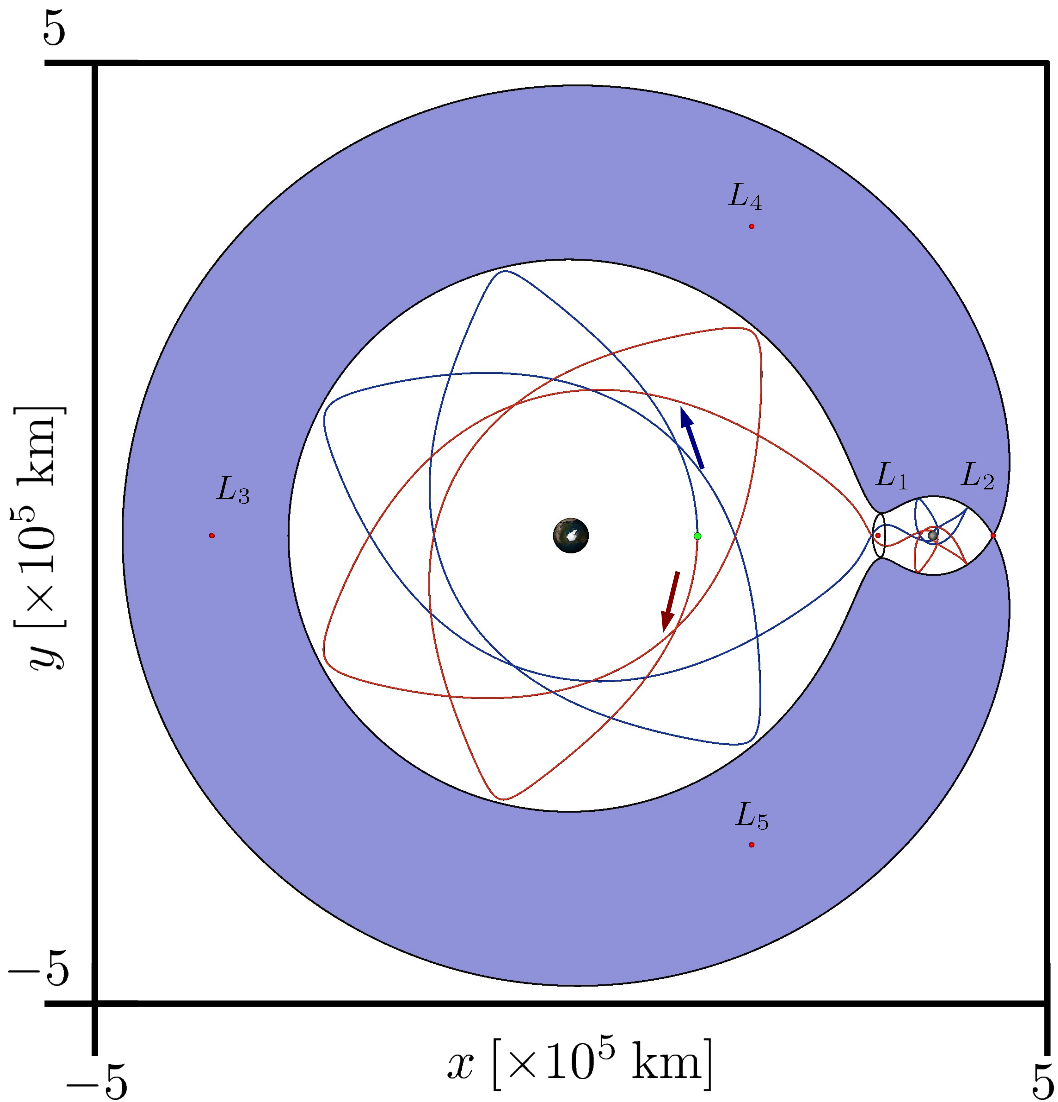


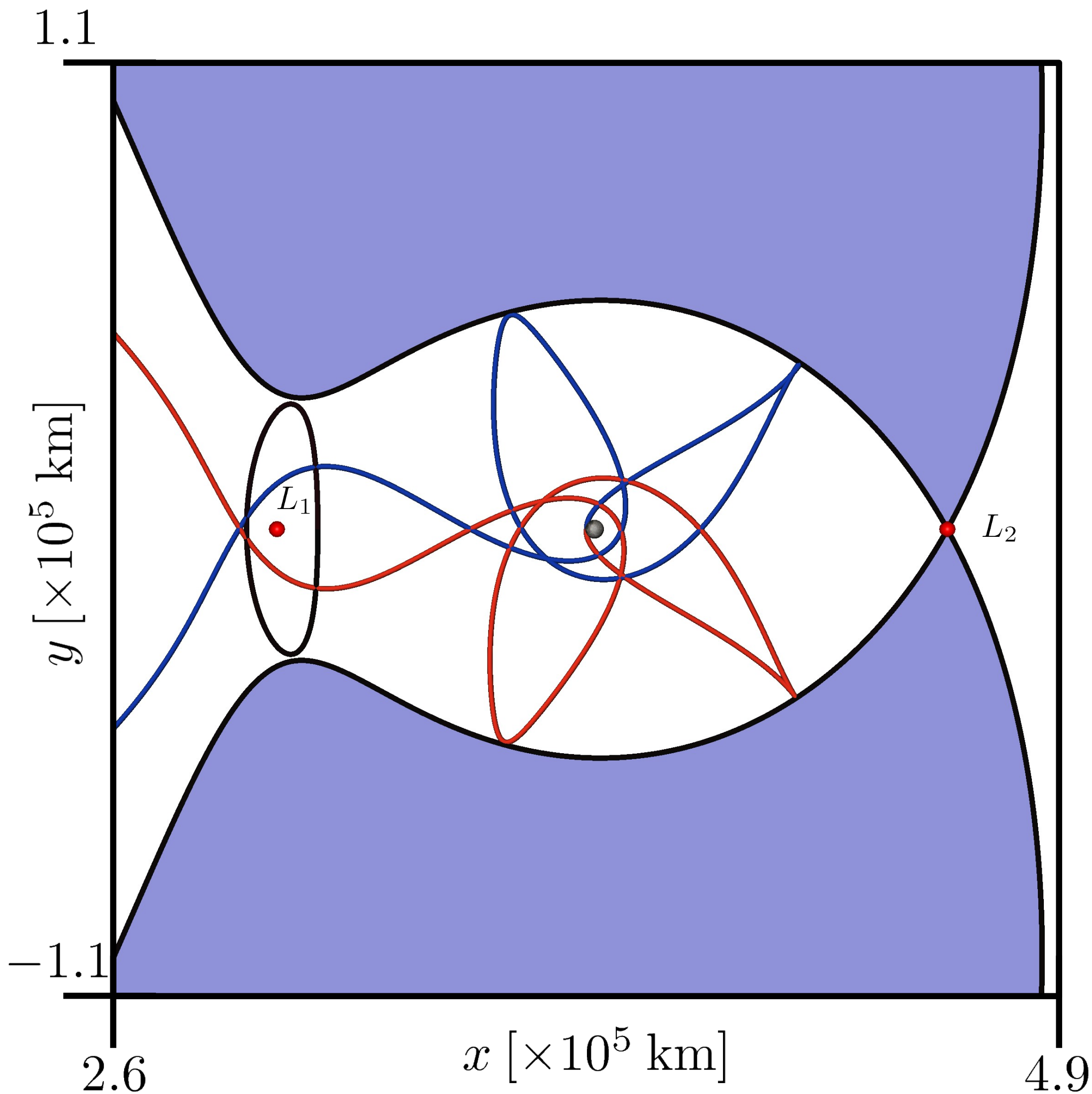
v_x
x


v_x
x

Ridge Point Extraction




5
−5
L_4
L_3
L_1
L_2
L_5
$y\ [\times 10^5\ \text{km}]$
$x\ [\times 10^5\ \text{km}]$
−5
5


1.1
−1.1
y [$\times 10^5$ km]
2.6
4.9
x [$\times 10^5$ km]
L_1
L_2

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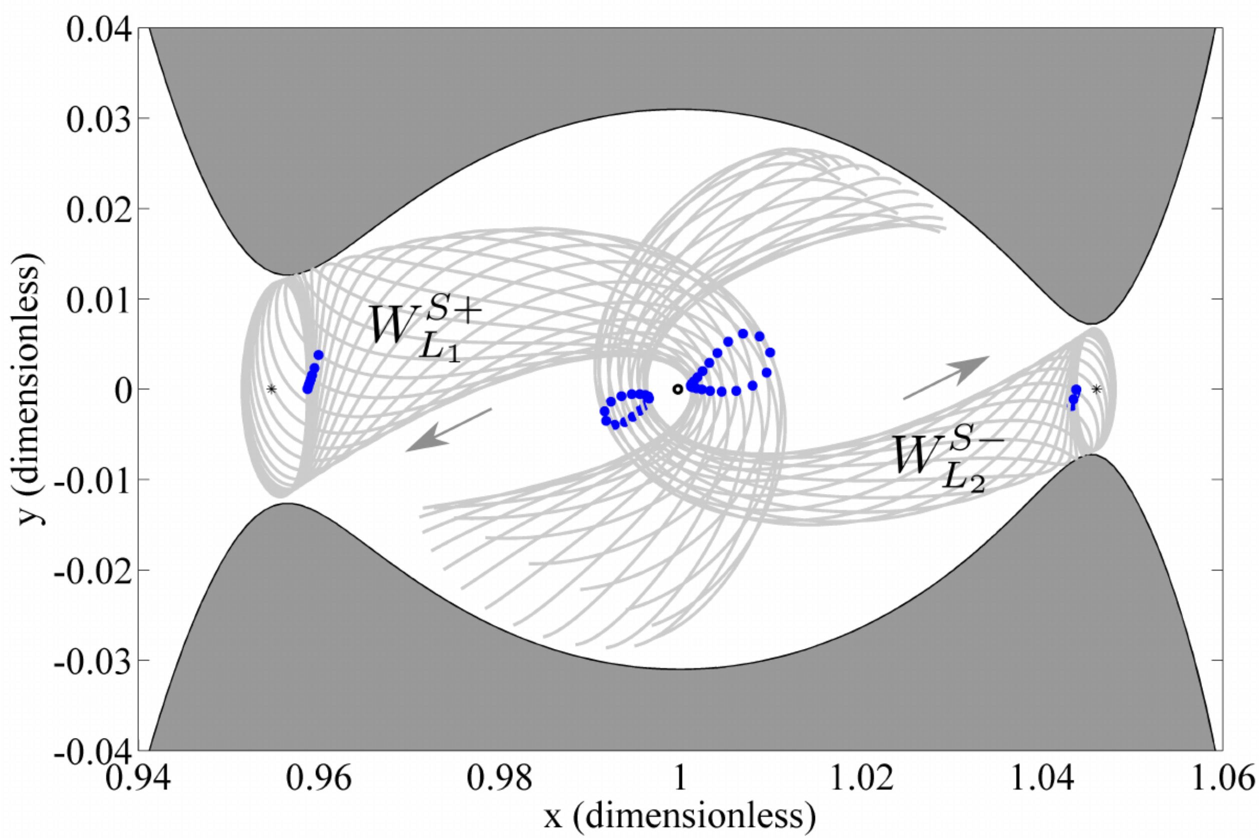
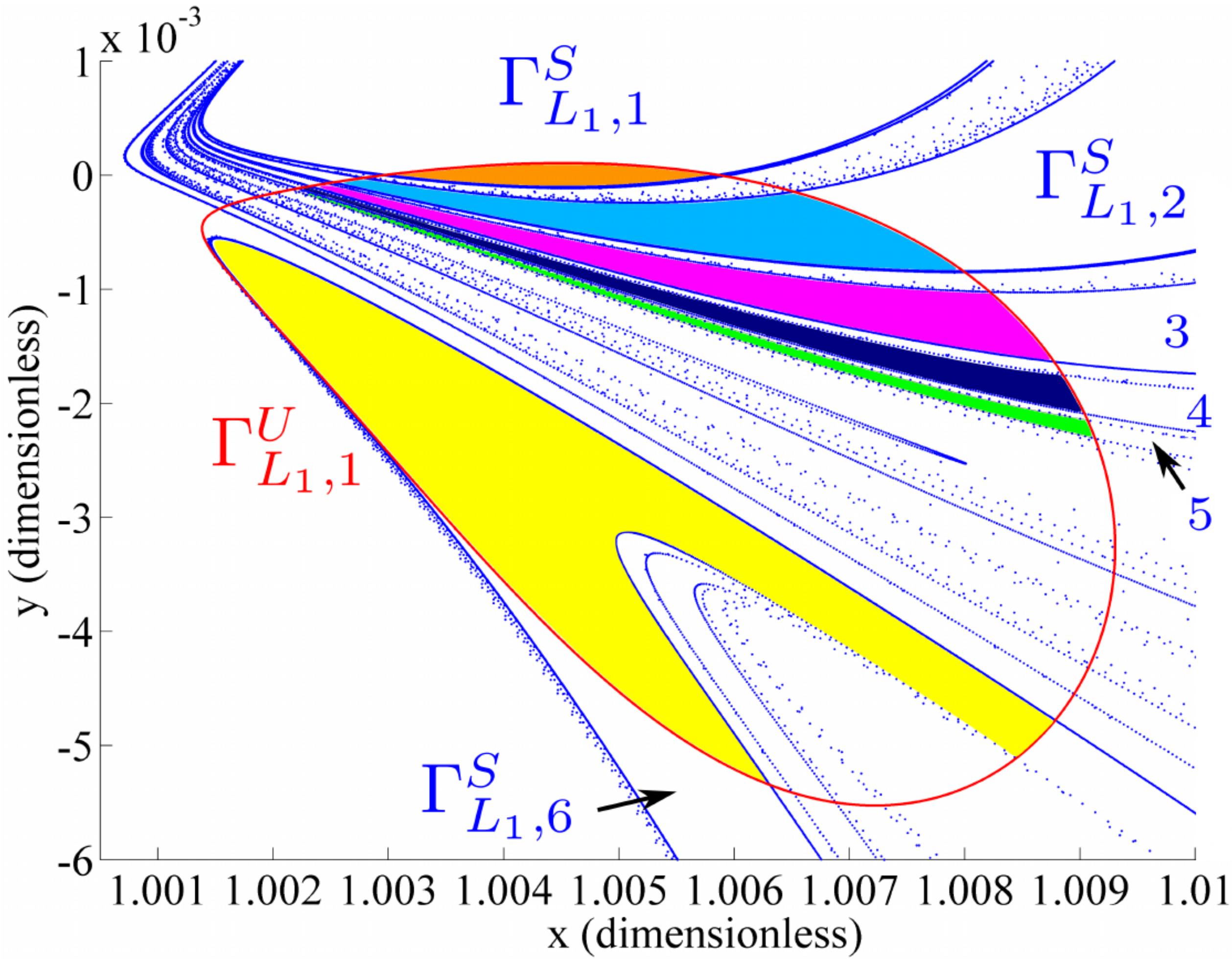
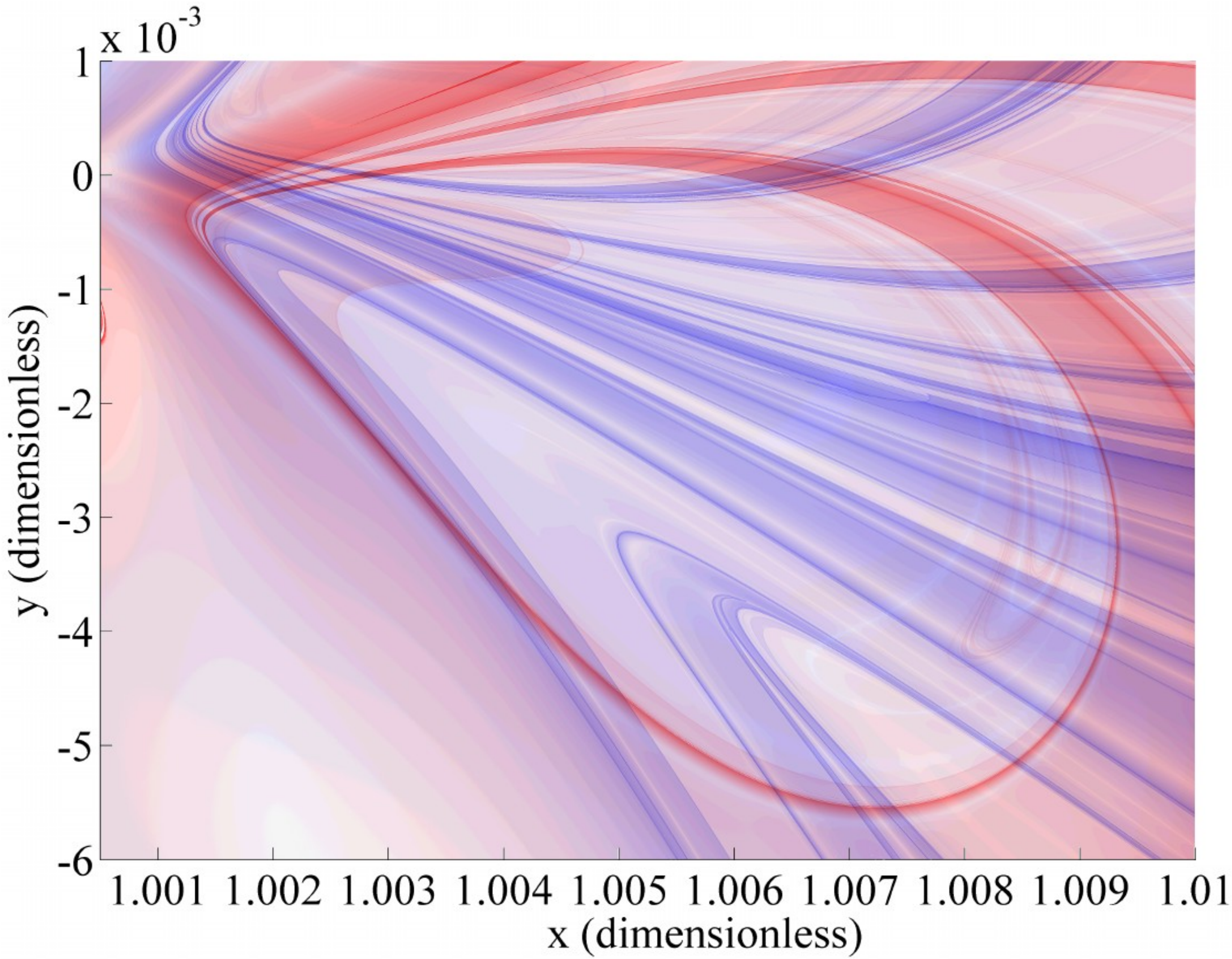
$\mathbf{x} : \dot{r} = 0, \ddot{r} \geqslant 0$

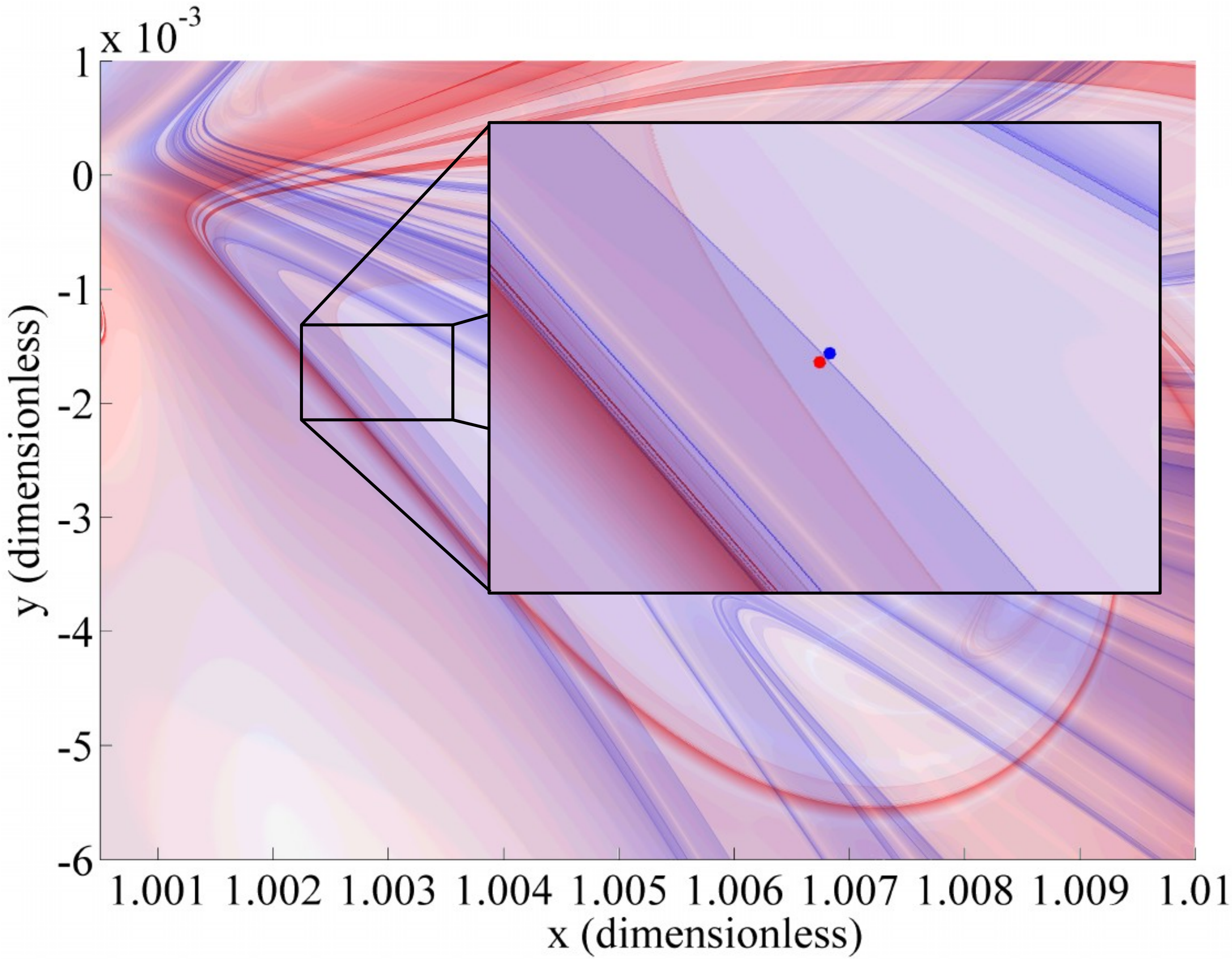


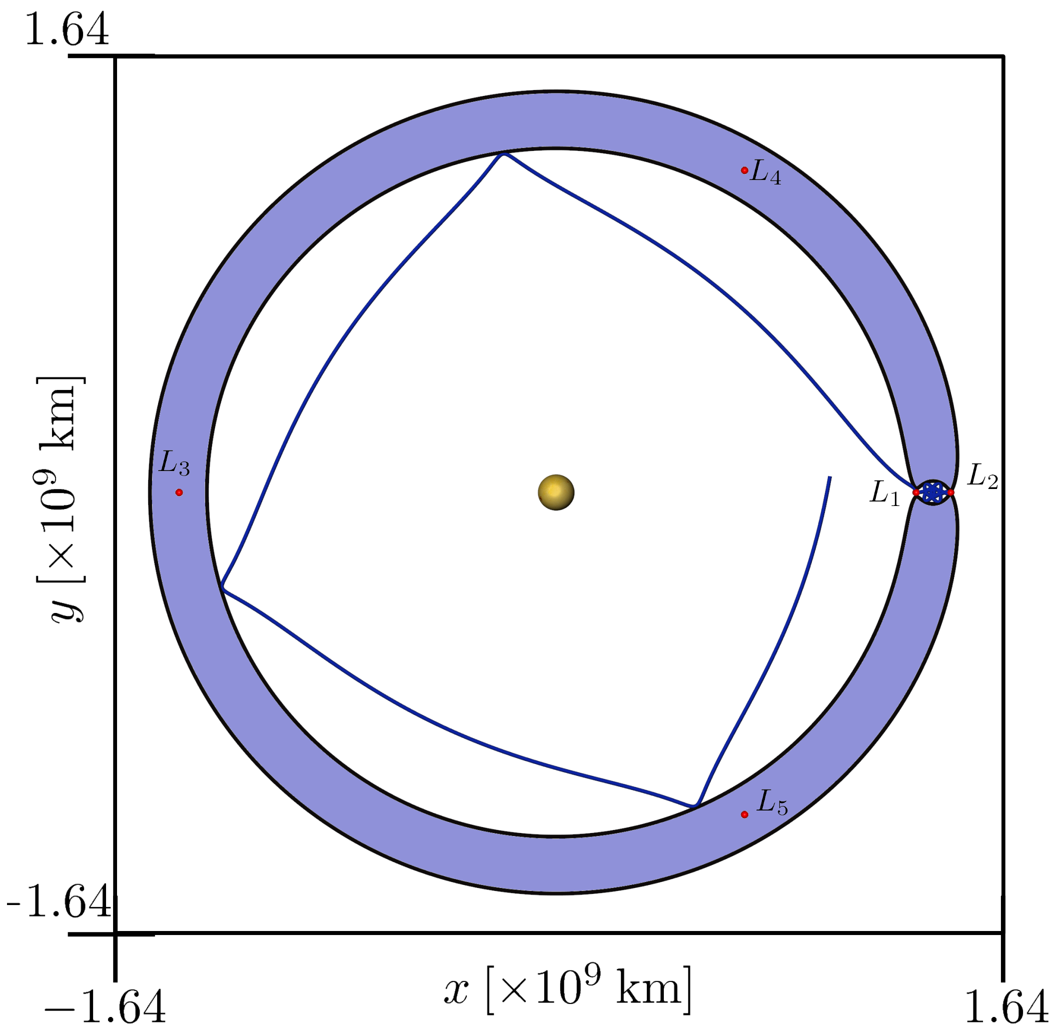
Source: A. Haapala, 2010

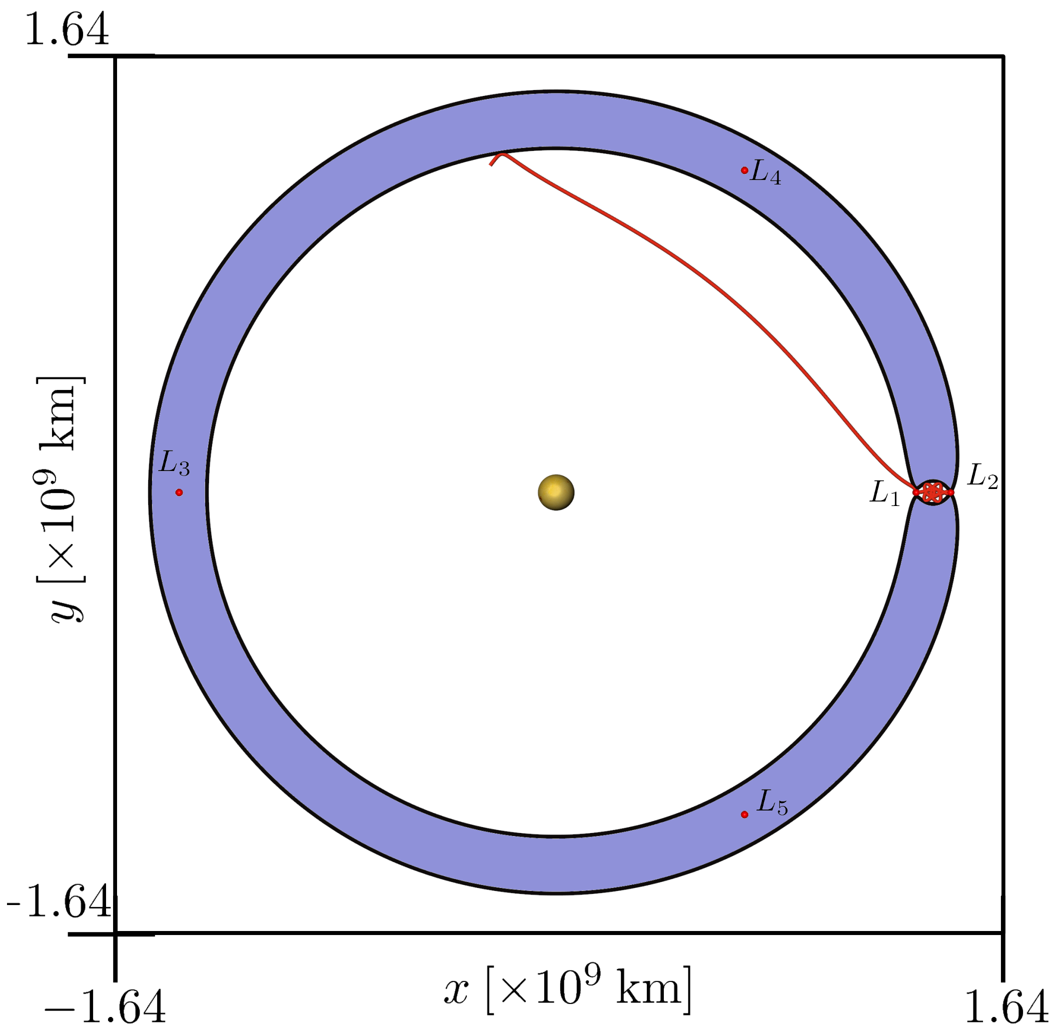


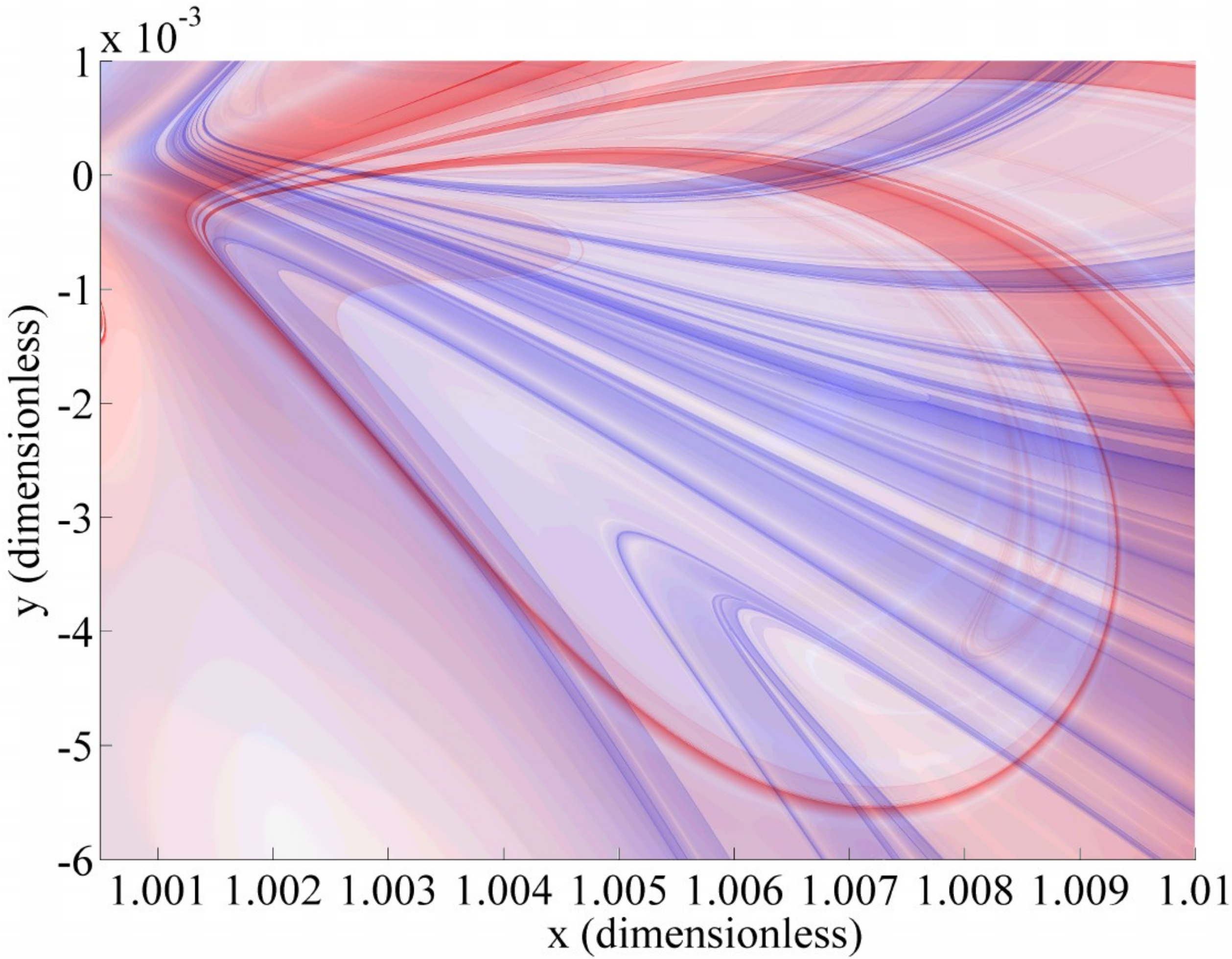
Source: A. Haapala, 2010


x 10^-3
1
0
-1
-2
-3
-4
-5
-6
y (dimensionless)
1.001 1.002 1.003 1.004 1.005 1.006 1.007 1.008 1.009 1.01
x (dimensionless)

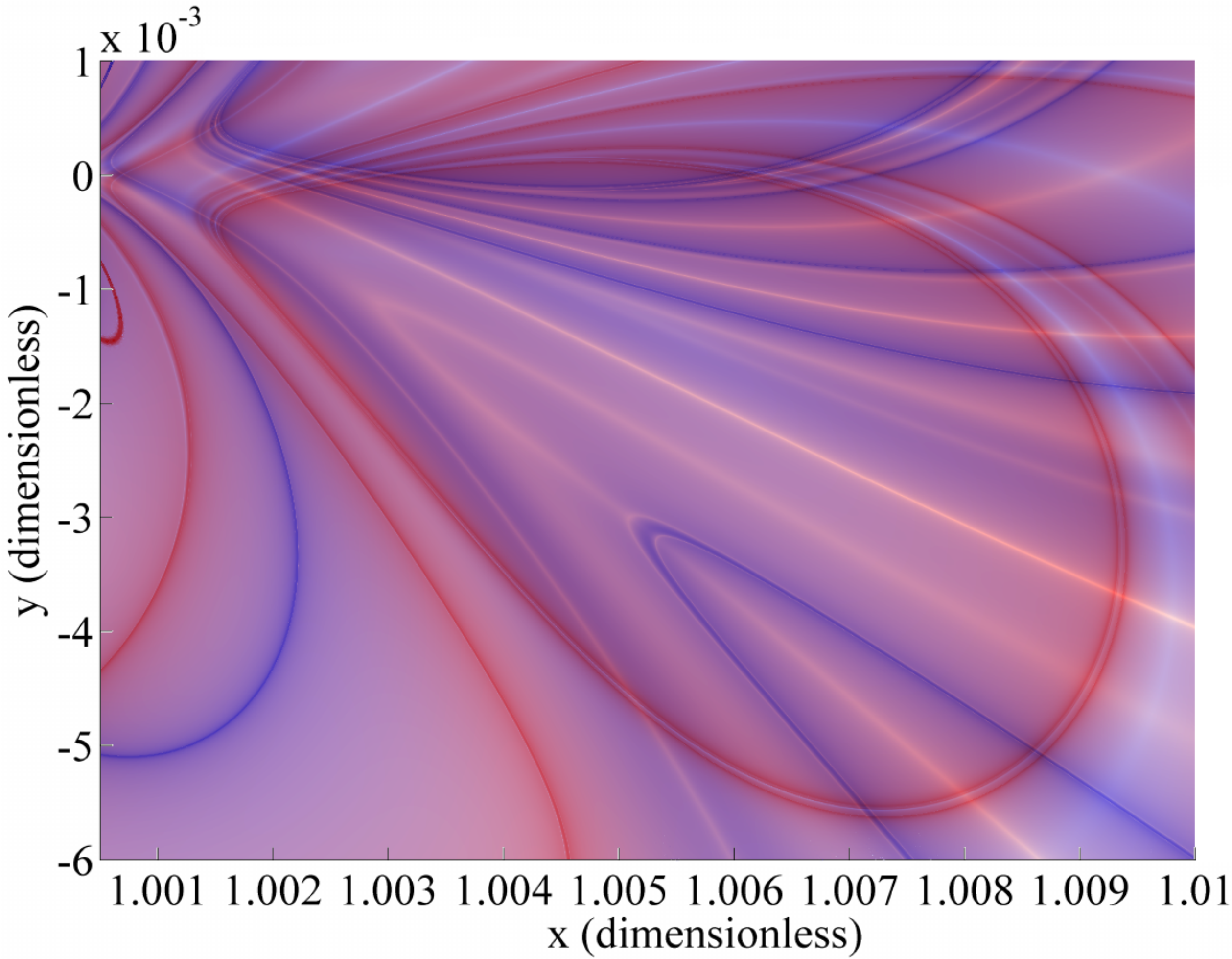

x 10^-3
1
0
-1
-2
-3
-4
-5
-6
y (dimensionless)
1.001 1.002 1.003 1.004 1.005 1.006 1.007 1.008 1.009 1.01
x (dimensionless)


1.64
-1.64
y [$\times10^9$ km]
L_4
L_3
L_1
L_2
L_5
-1.64
x [$\times10^9$ km]
1.64

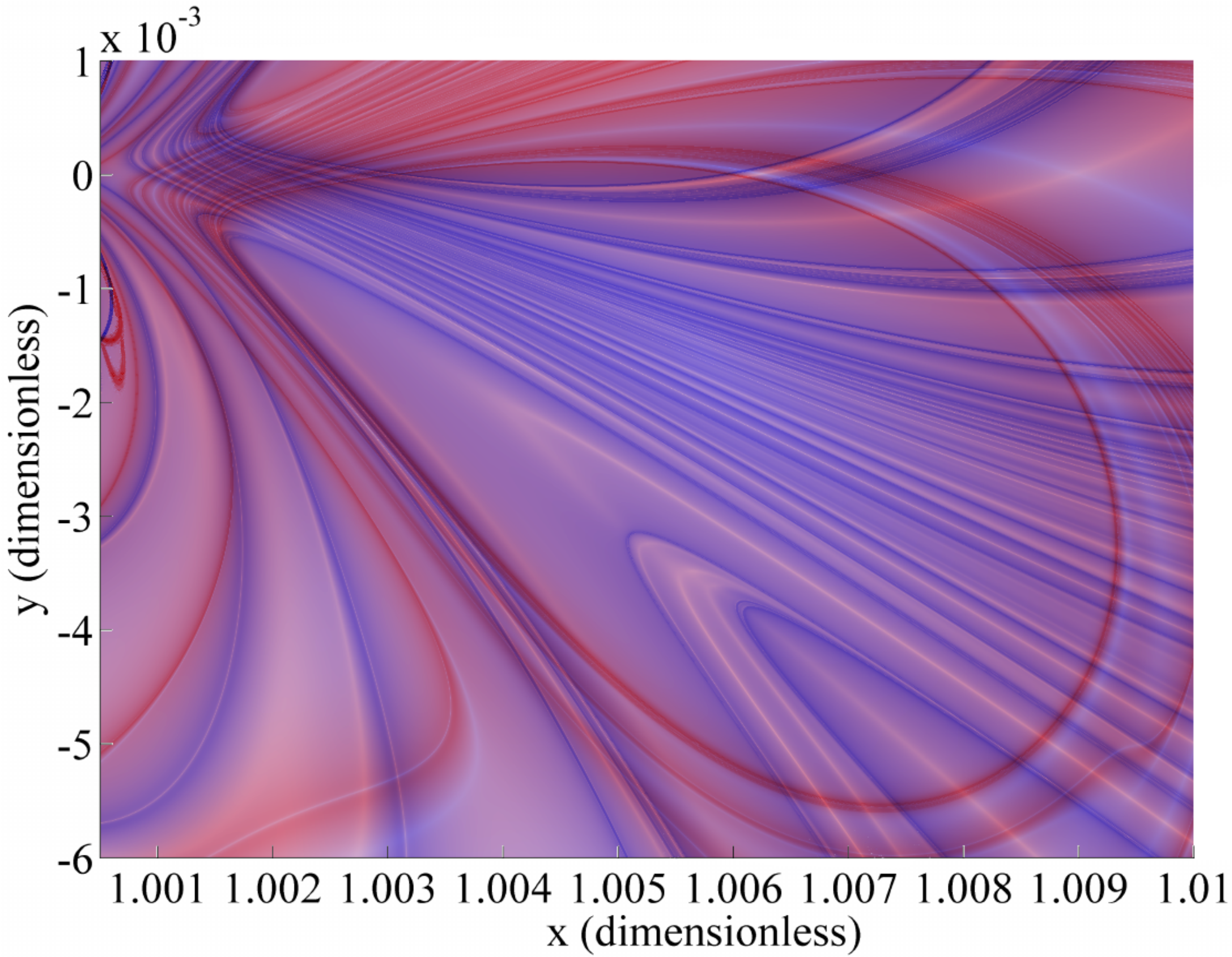

1.64
-1.64
y [$\times 10^9$ km]
L_3
L_4
L_1
L_2
L_5
-1.64
x [$\times 10^9$ km]
1.64


x 10^-3
1
0
-1
-2
-3
-4
-5
-6
y (dimensionless)
1.001 1.002 1.003 1.004 1.005 1.006 1.007 1.008 1.009 1.01
x (dimensionless)

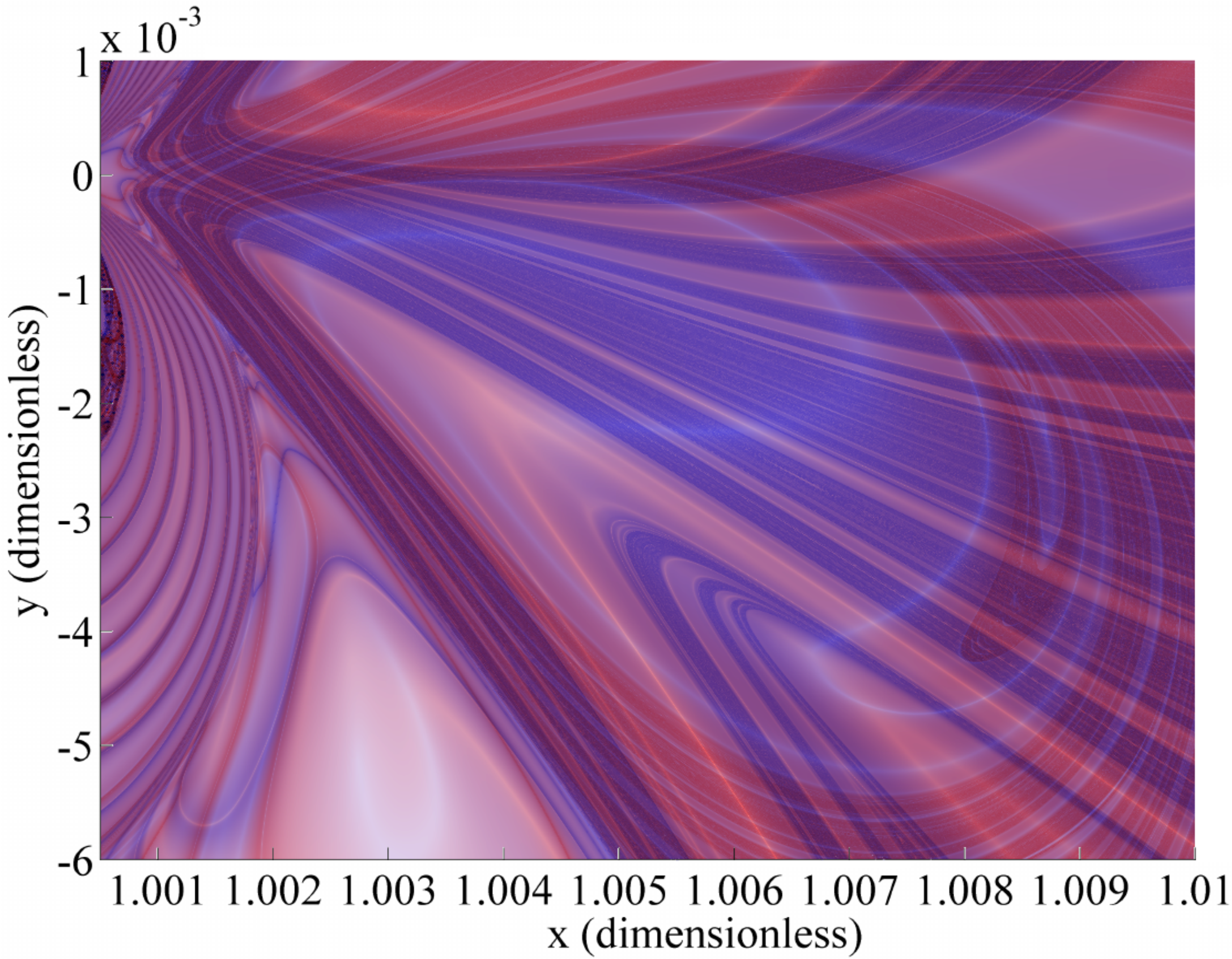
Periapse



Stroboscopic: 5 nondimensional time steps



Stroboscopic: 10 nondimensional time steps



Stroboscopic: 50 nondimensional time steps

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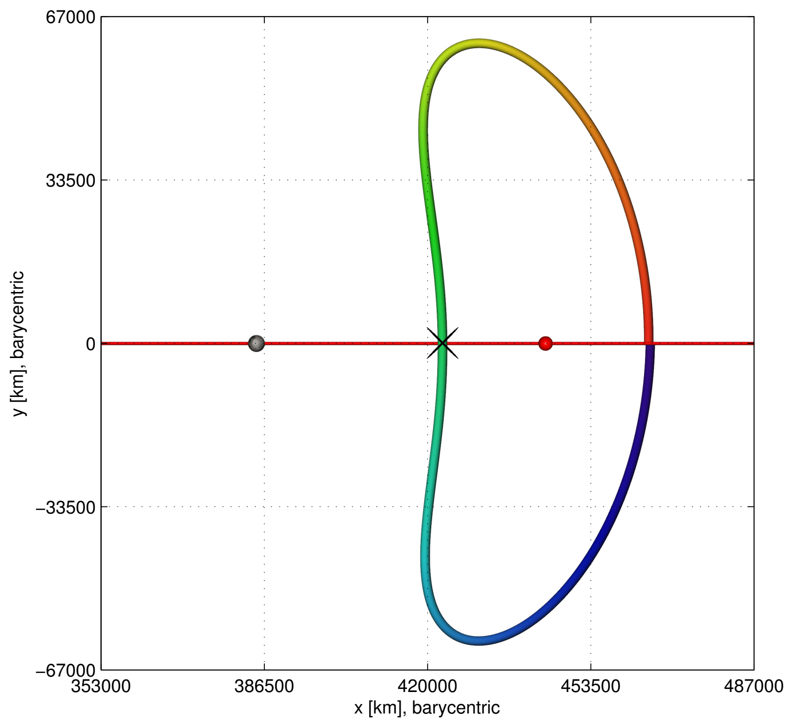
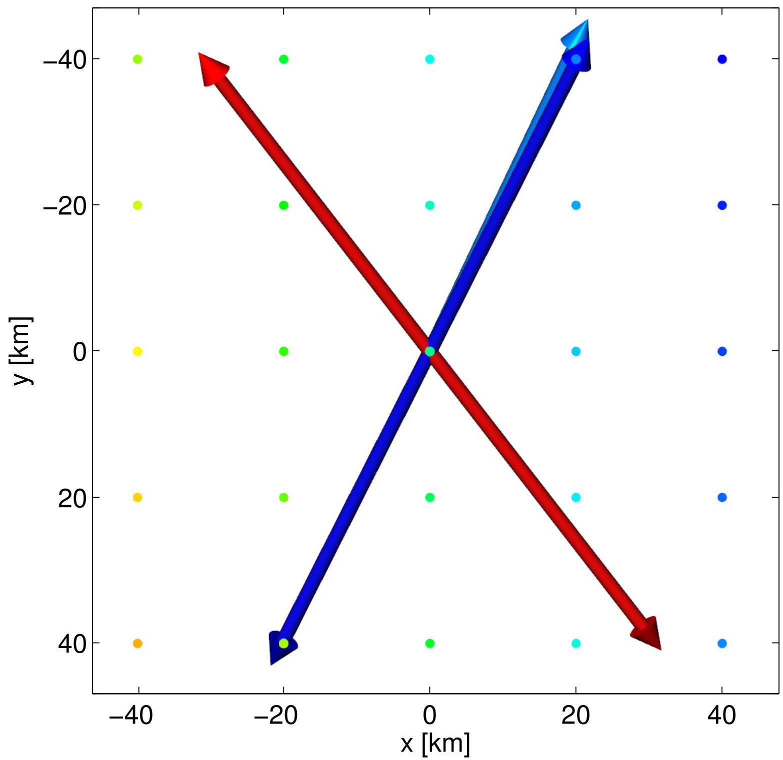
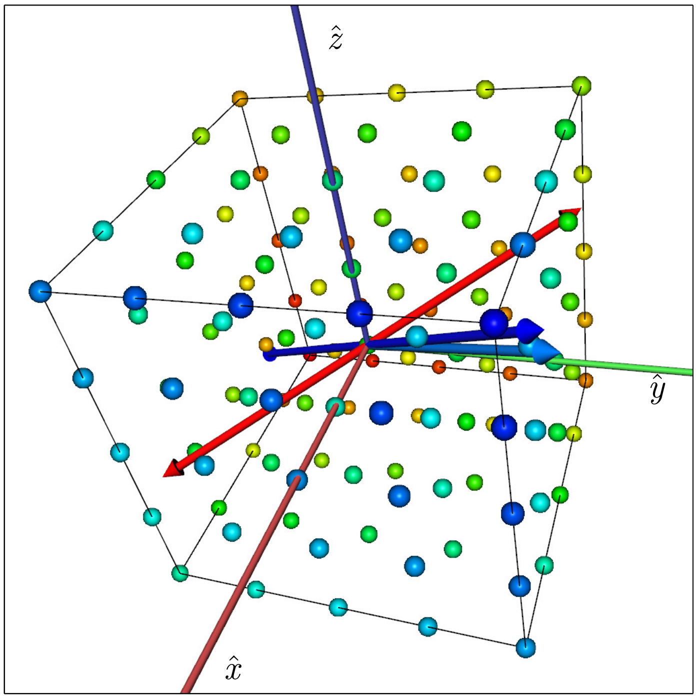
- November 17, 2010 08:45:00 UTC
- Maneuver point between the Moon and L_2 near the x axis
- Here, evolved forward and backward to the x axis
- Generate forward FTLE about the maneuver point in position space
- Establish stable/unstable directions from monodromy matrix
- Pavlak and Howell, Folta et al. $\rightarrow$ optimal maneuver $\sim$ stable direction



Unstable Directions,
Stable Directions,
Maneuver Direction





Space between grid points:
20km in position directions

Maneuver FTLE Summary

SKM	L_1/L_2	Δt	Stable Alignment	FTLE Consistent	S/U Sep OK
P1-03	L_2	3.50	Y	Y	N
P1-04	L_2	3.40	Y	Y	Y
P1-05	L_2	3.70	∼Y	Y	N
P1-06	L_2	3.50	Y	Y	Y
P1-07	L_2	3.40	∼Y	Y	N
P1-08	L_2	3.57	Y	Y*	Y
P1-09	L_2	3.70	Y	Y	N
P1-10	L_2	3.50	Y	Y	Y
P1-11	L_2	3.60	Y	N	Y
P1-16	L_1	2.92	Y	Y	Y
P1-17	L_1	3.20	Y	Y*	Y
P1-18	L_1	3.20	Y	N	Y
P1-19	L_1	3.00	Y	Y	Y
P1-20	L_1	2.90	Y	Y	Y
P1-21	L_1	3.05	Y	Y*	Y
P1-22	L_1	3.30	Y	Y	Y
P1-23	L_1	3.10	Y	Y	Y
P1-25	L_1	2.97	Y	Y	Y
P1-26	L_1	3.25	Y	Y	Y
P1-27	L_1	3.20	Y	Y	Y
P1-28	L_1	2.95	Y	N	Y
P1-29	L_1	2.95	Y	N	Y
P1-30	L_1	3.20	Y	Y	Y
P1-31	L_1	3.20	Y	Y*	Y
P1-32	L_1	2.95	Y	Y	Y
P1-33	L_1	2.95	Y	Y	Y
P1-34	L_1	3.20	Y	Y*	Y

- Demonstrate and Establish
  - FTLE/LCS correlation
  - Periapse maps and the FILE
- Validate
  - Model fidelity impact on flow structures
  - Maneuver direction analysis
- Develop and Extend
  - Flow control segments
  - Trajectory design example in a nonautonomous system

Extension of ‘control segment’ strategy (Schroer & Ott and Grebow)

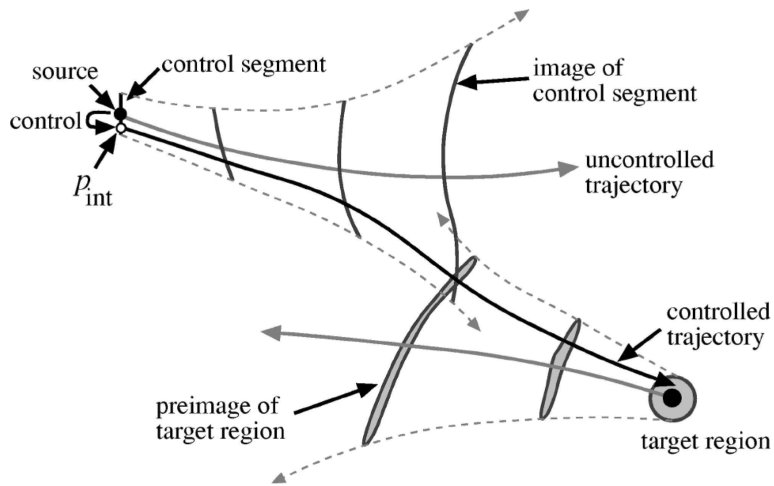
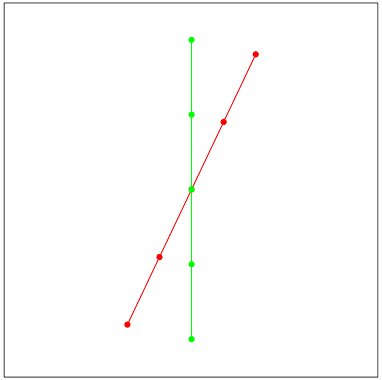
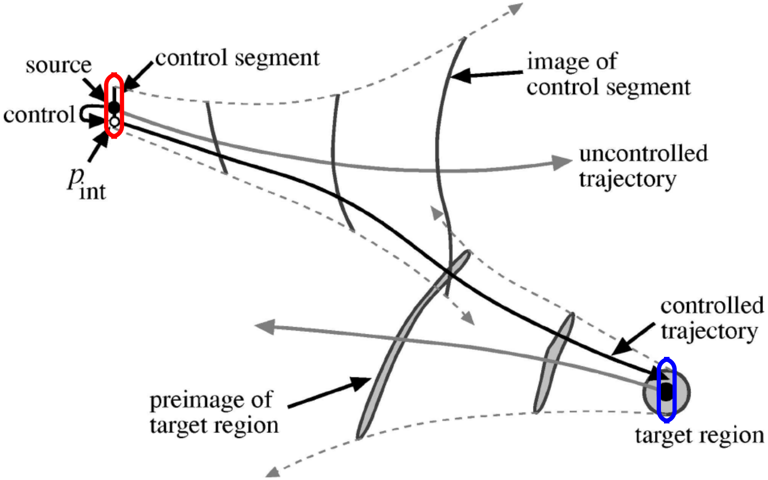


Figure: Schroer & Ott (with permission)

Extension of ‘control segment’ strategy (Schroer & Ott and Grebow)





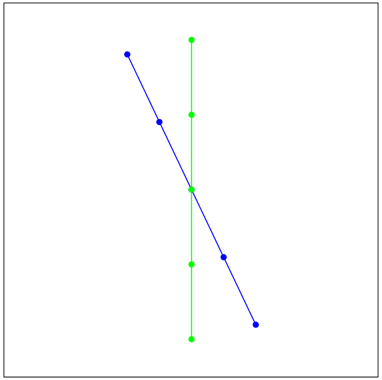
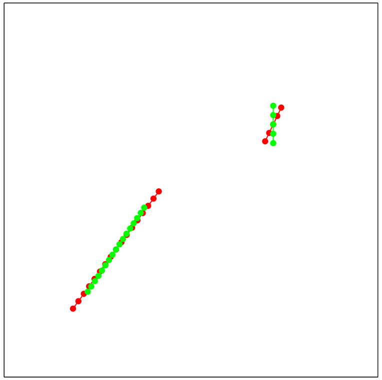


Figure: Schroer & Ott (with permission)

Extension of ‘control segment’ strategy (Schroer & Ott and Grebow)



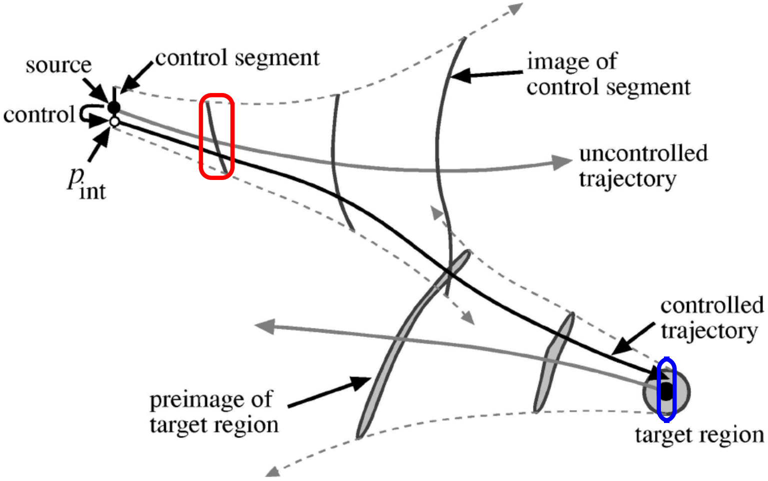
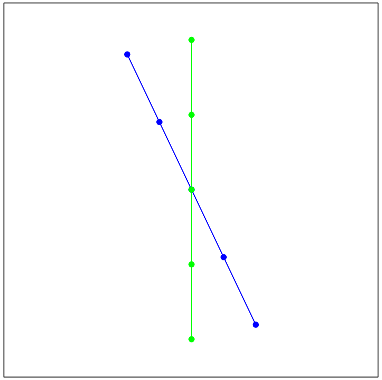
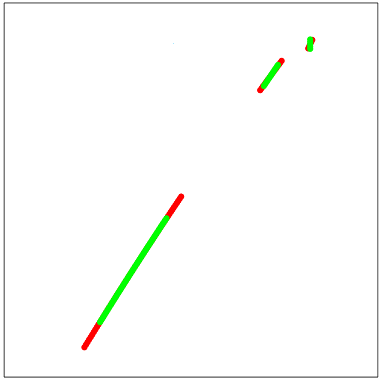
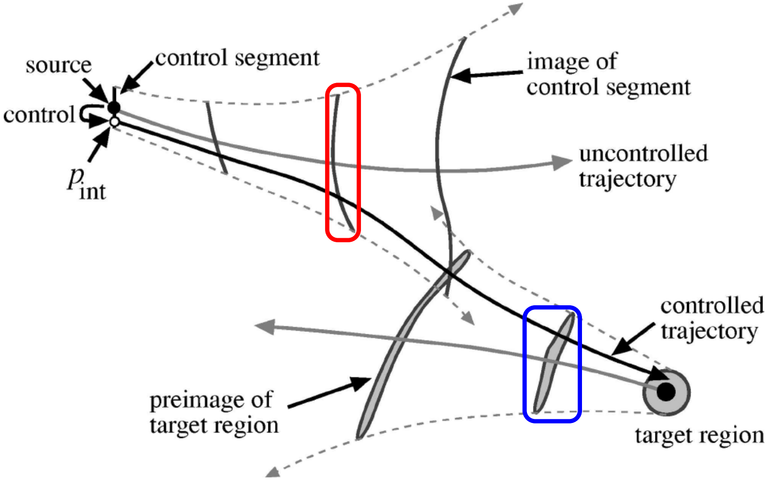


Figure: Schroer & Ott (with permission)



Extension of 'control segment' strategy (Schroer & Ott and Grebow)





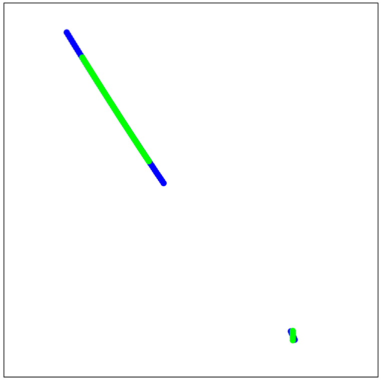
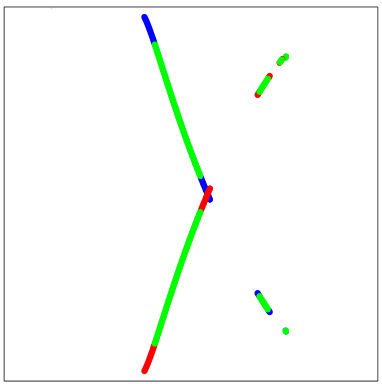
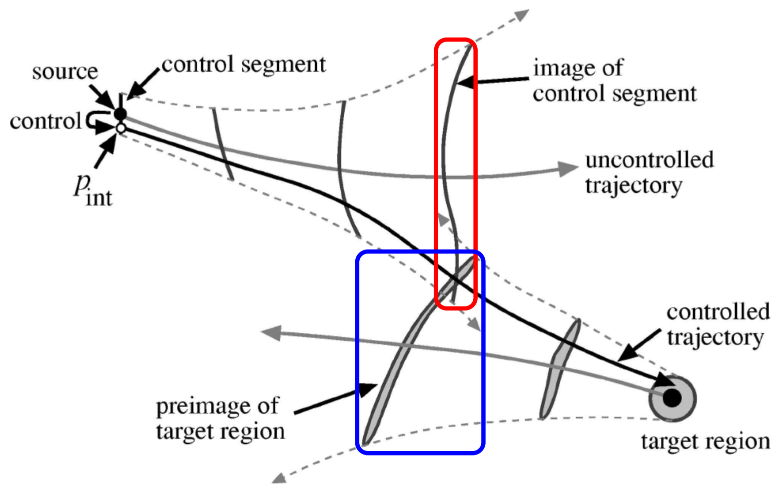


Figure: Schroer & Ott (with permission)

Flow Control Segments (FCS)

Extension of ‘control segment’ strategy (Schroer & Ott and Grebow)





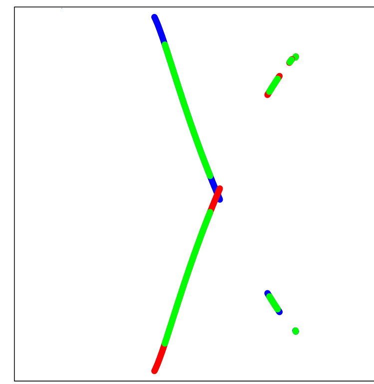
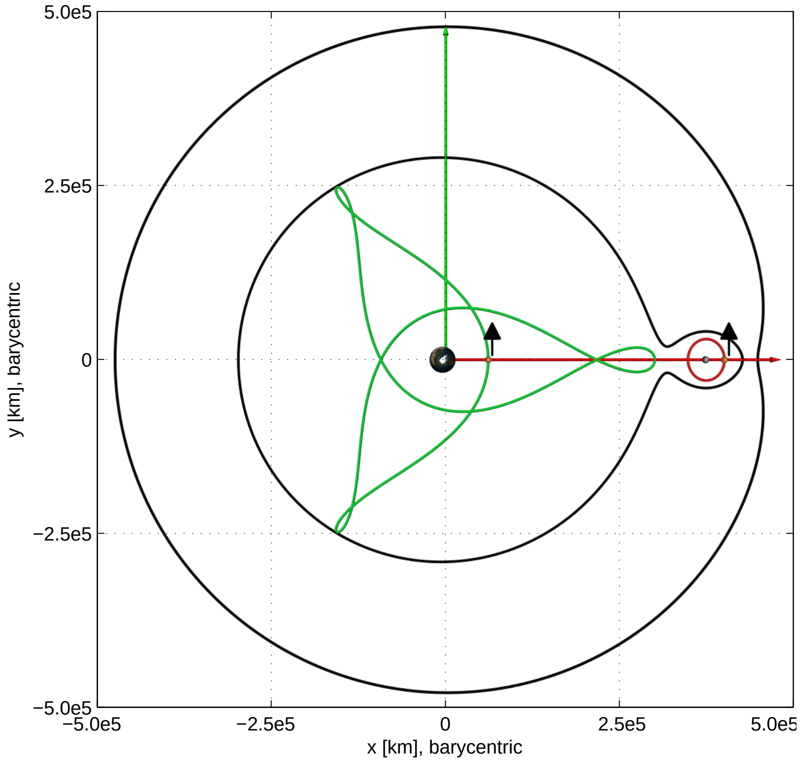
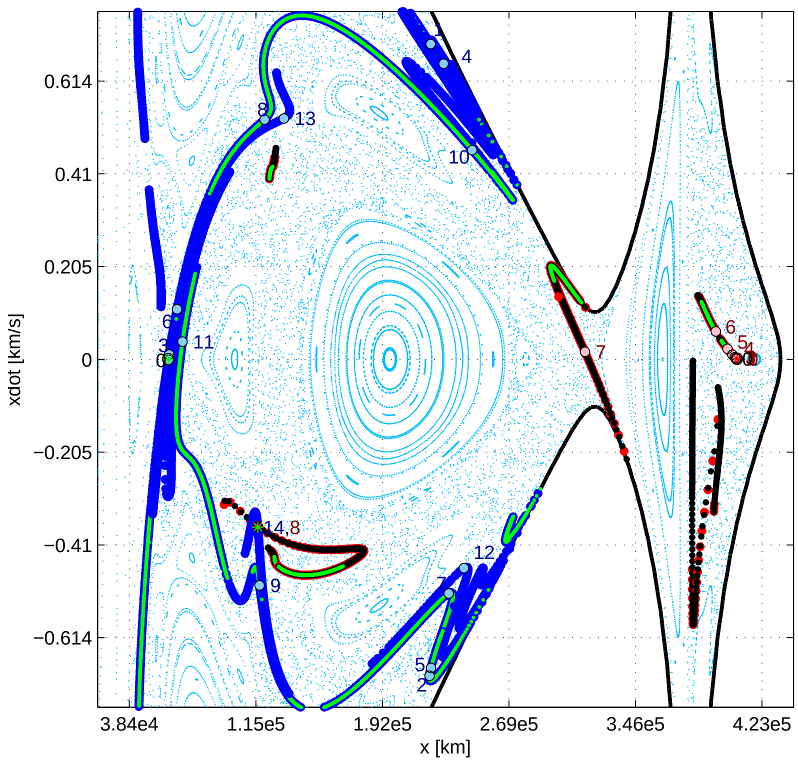


Figure: Schroer & Ott (with permission)

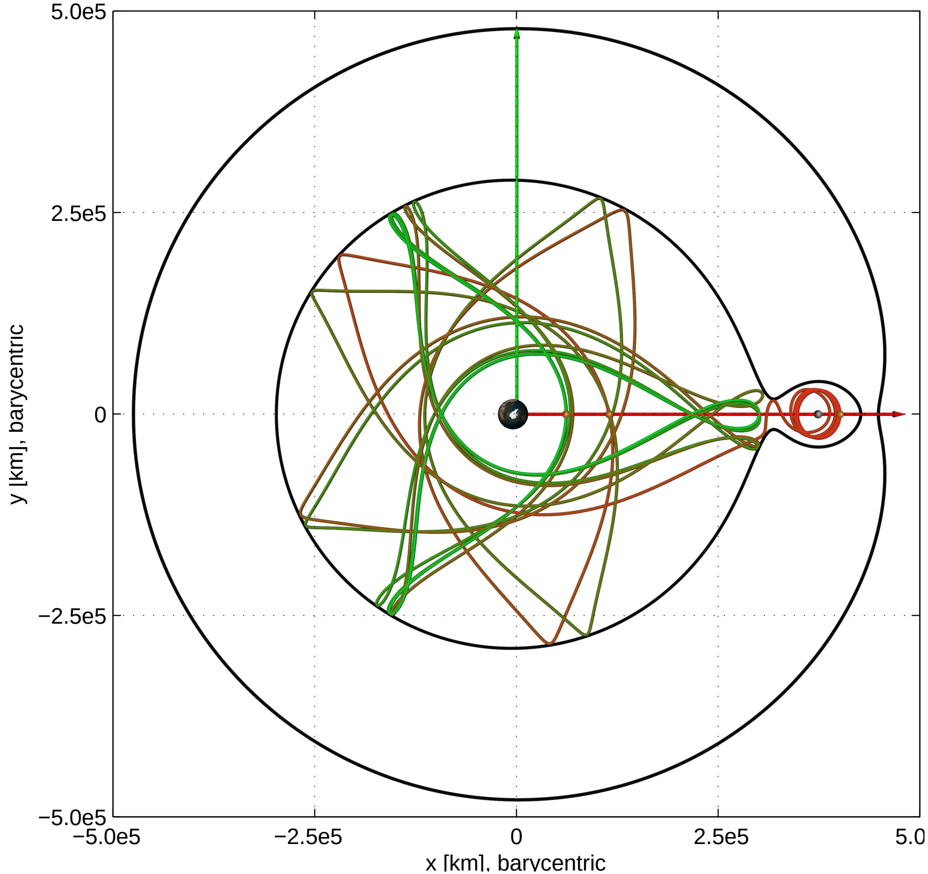
Objective: Connect two chaotic CRP orbits using control segments

- Validation of FCS approach
- Illustration in lower complexity system

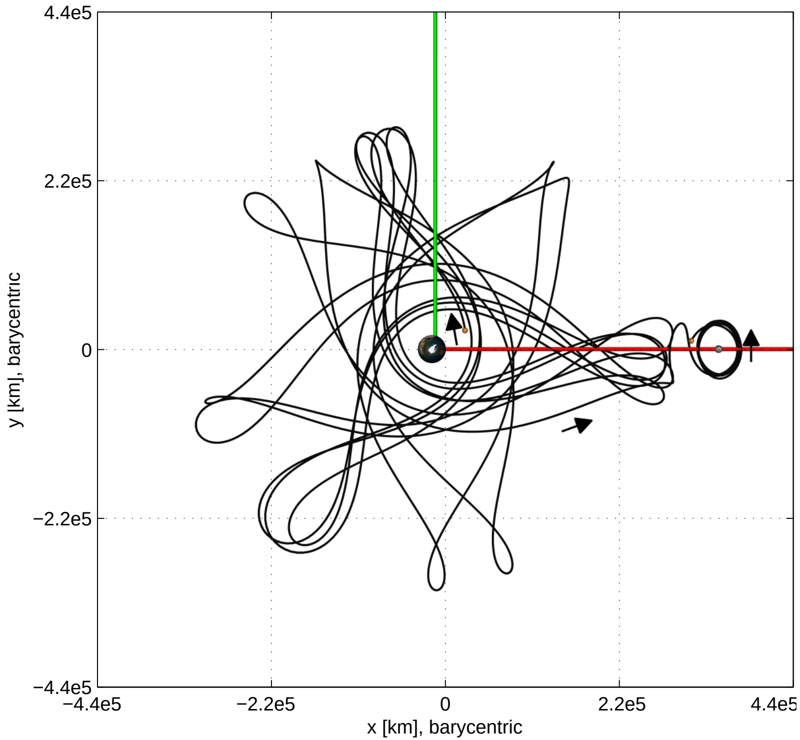




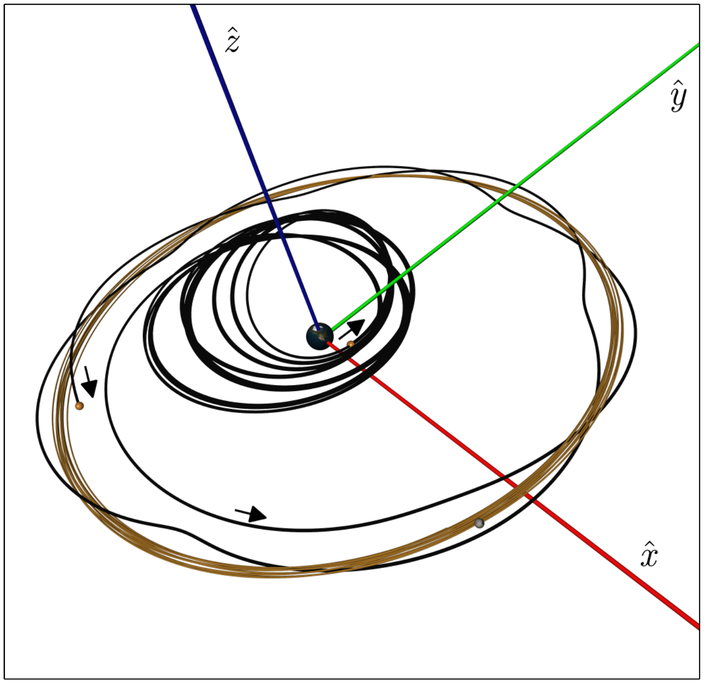
- Connection after 14 forward and 8 backward iterations (fewer)
- Time of flight: 265 days (other studies: ∼290 days)
- Maneuver cost: 3.03 m/s (3.23 m/s, Schroer & Ott; 1.85 m/s, Grebow)
- Validation of FCS approach



- Transitioned with Adaptive Trajectory Design (Haapala, Pavlak)
- Epoch: 09Dec13 19:45 UTC
- Out-of-plane excursions up to 2000 km

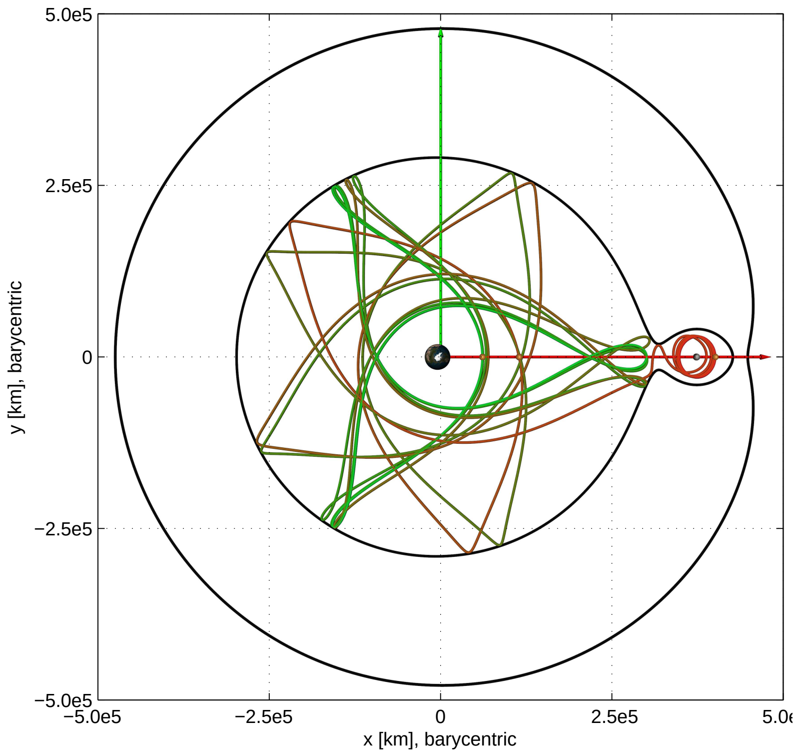


Earth-Moon Rotating

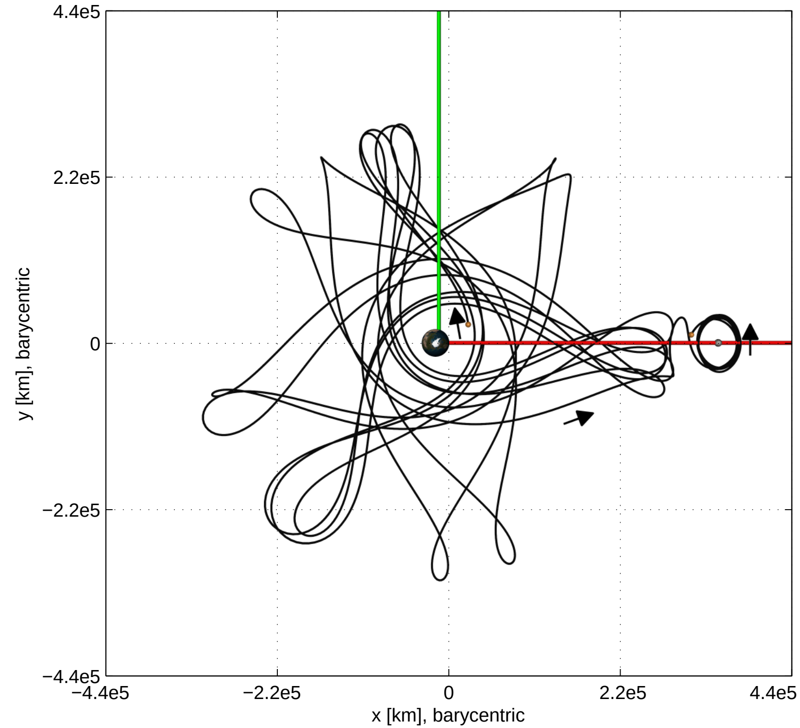


Earth-Centered Inertial

- Further validation of solutions from medium-fidelity models
- Identification of trajectory behavior-shifting mechanisms
- Rigorous structure identification in higher-dimensional systems
- Analysis of regions bounded by parametrized strainlines

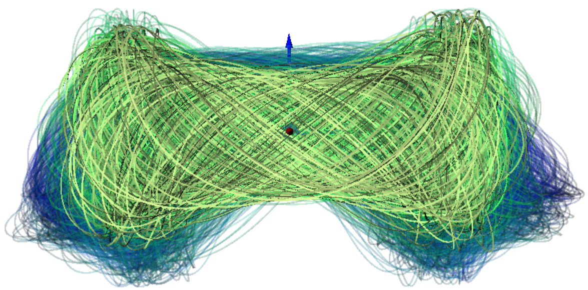


FCS Identified CRP Solution

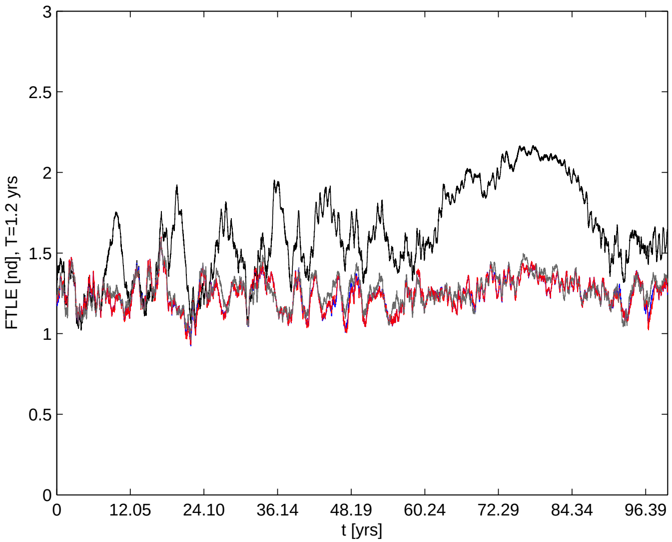


Ephemeris Transitioned Solution

- Further validation of solutions from medium-fidelity models
- Identification of trajectory behavior-shifting mechanisms
- Rigorous structure identification in higher-dimensional systems
- Analysis of regions bounded by parametrized strainlines

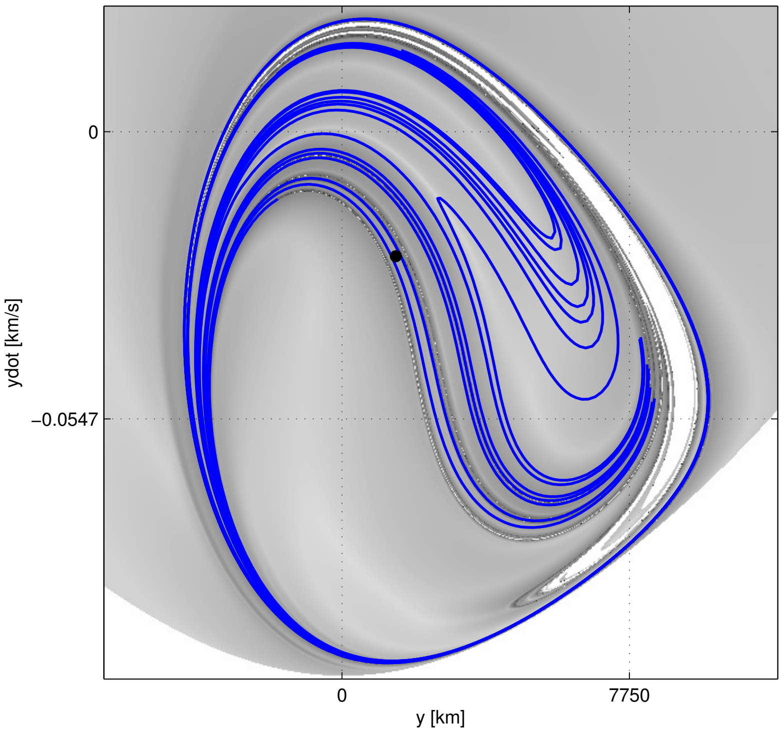


TESS Orbit Long-term Evolution

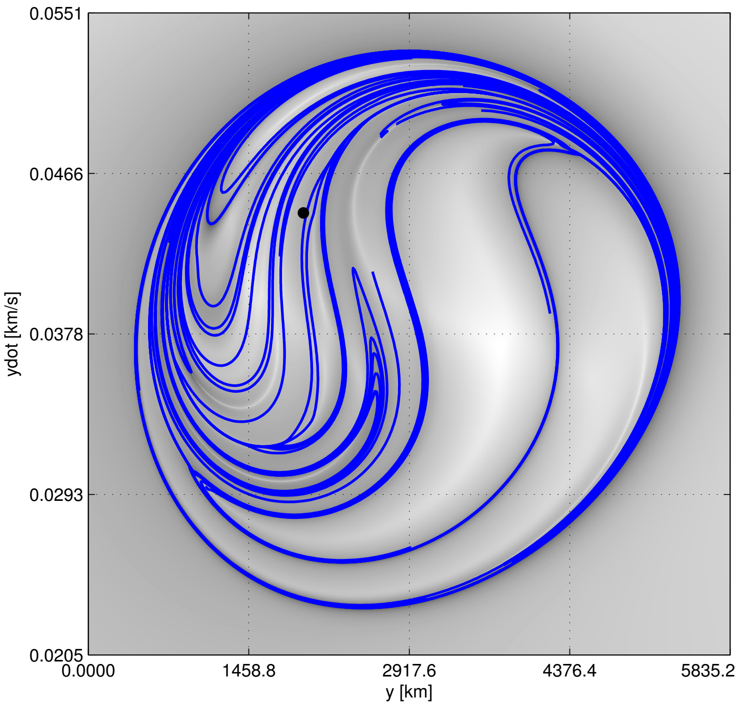


FTLE Model Component Analysis

- Further validation of solutions from medium-fidelity models
- Identification of trajectory behavior-shifting mechanisms
- Rigorous structure identification in higher-dimensional systems
- Analysis of regions bounded by parametrized strainlines



Strainlines on Titania map



Strainlines on Oberon map

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